Coursework 3 - Quantum mechanics
Deadline: 20 March 2008

1. The spin-dependent Hamiltonian of an electron-positron system in the presence of a uniform magnetic field in the z direction can be written as

$$\hat{H} = A\mathbf{S}^{(e^-)} \cdot \mathbf{S}^{(e^+)} + \left(\frac{eB}{mc}\right)\left(S_z^{(e^-)} - S_z^{(e^+)}\right).$$

Suppose the spin function of the system is given by $\chi_\uparrow^{(e^-)}\chi_\downarrow^{(e^+)}$,i.e., the electron is in a spin-up state and the positron in a spin-down state. Is this an eigenfunction of the system

   (a) For $A = 0$, $B \neq 0$?

   (b) For $A \neq 0$, $B = 0$?
   If it is, what is the energy eigenvalue?If it is not, what is the expectation value of $\hat{H}$?

2. The wave function of a particle subjected to a spherically symmetric potential $V(r)$ is given by

$$\psi(\mathbf{r}) = (x + y + 3z)f(r).$$

   (a) Is ψ an eigenfunction of L^2?If so, what is the l-value?If not, what are the possible values on may obtain when L^2 is measured?

   (b) What are the probabilities for the particle to be found in various m_l states?

3. Two particles of spin 1/2 have spin operators $\mathbf{S}_1$ and $\mathbf{S}_2$. The two-particle system is described by the singlet and triplet wave functions

$$\chi_{0,0} = \frac{1}{\sqrt{2}}\left[\alpha(1)\beta(2) - \beta(1)\alpha(2)\right]$$

and

$$\begin{aligned}
\chi_{1,1} &= \alpha(1)\alpha(2) \\
\chi_{1,0} &= \frac{1}{\sqrt{2}}\left[\alpha(1)\beta(2) + \beta(1)\alpha(2)\right] \\
\chi_{1,-1} &= \beta(2)\beta(1),
\end{aligned}$$

respectively. Find the expectation value of the product $\mathbf{S}_1 \cdot \mathbf{S}_2$ in the singlet spin state.

4. The spin properties of a system of two spin 1/2 particles are described by the singlet and triplet wave functions stated in the previous question. If a third particle of spin 1/2 is added to the system, show that the total spin quantum number of the particle can be s=3/2 (a quartet sate), or s=1/2 (a doublet state), and find the corresponding spin functions.
   Hint: treat the original two-particle system as a single entity of either spin 0 or 1 and use the same considerations as in the case $\mathbf{J} = \mathbf{L} + \mathbf{S}$ discussed in class.

5. A particle of mass m is placed in an infinite one-dimensional potential well of width a

$$V(x) = \begin{cases} 0, -a/2 \leq x \leq a/2 \\ +\infty \quad \text{elsewhere} \end{cases}$$

   Assume that this particle, of charge q, is subject to a uniform electric field E, with the corresponding perturbation being

$$W = qEx.$$

   Let ε_1 and ε_2 be the corrections to first and second-order in E for the energy of the ground state.

   (a) Show that ε_1 is zero. Did you expect that?Why or why not?
   (b) Give the expression for ε_2 in form of a series, whose terms are to be calculated in terms of $q, E, m, a, \hbar$
   (c) Give a lower bound for ε_2, obtained by retaining only the principal term of the series.

You will need to employ the eigenfunctions $\psi_n(x) = \sqrt{2/a}\cos(n\pi x/a)$, n odd, and $\psi_n(x) = \sqrt{2/a}\sin(n\pi x/a)$, n even, and the eigenenergies

$$E_n = \frac{n^2\pi^2}{2ma^2}$$