# Experimental Evidence for General Relativity

# Experimental Tests of GR

- Mass Equivalence (Eotvos, Dicke)
- Lunar Ranging
- Time Dilation (Pound and Rebka, GPA, stellar emission lines)
- Gravity Probe B (Geodetic effect, Frame Dragging)
- Shapiro Effect
- Precession of the Perihelion of Mercury
- Deflection of Light (Solar eclipses)
- Black Holes and Accretion (Orbits, Accretion luminosity)
- Distance Measures in Cosmology ($\Lambda$CDM)
- Binary Pulsars and Gravitational Waves

# Mass Equivalence

- Fundamental to GR is the equivalence of inertial and gravitational mass

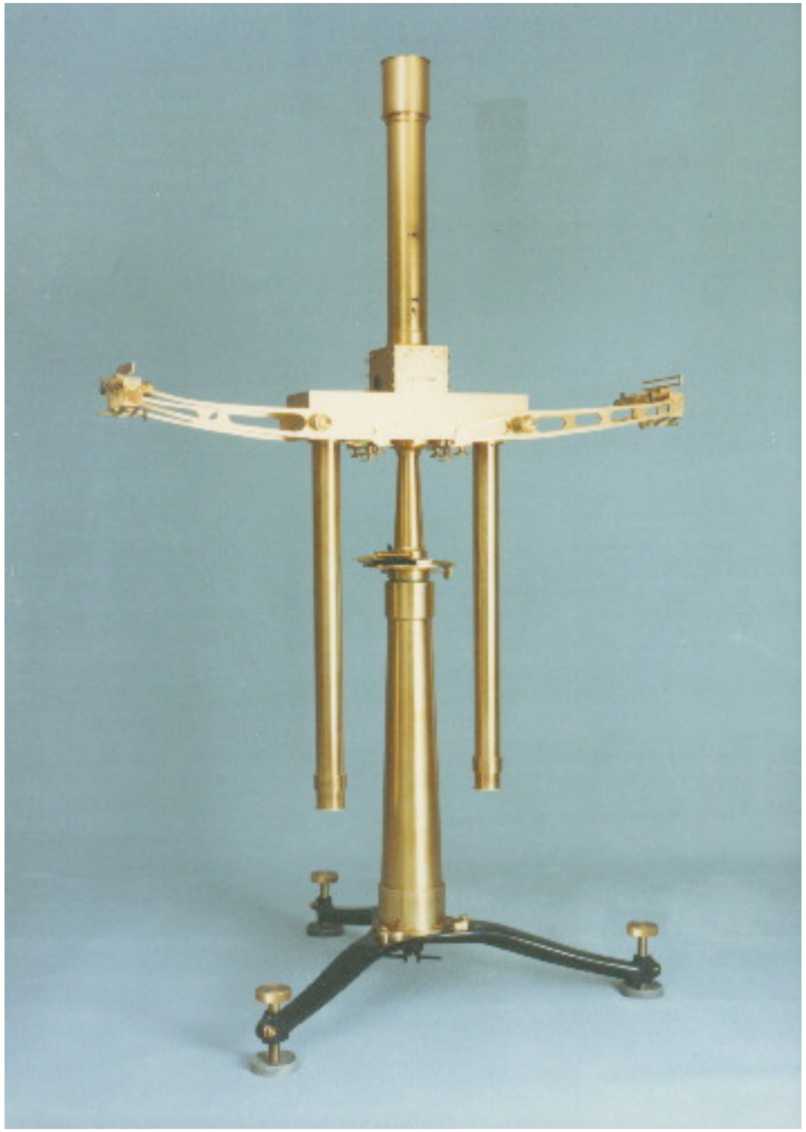
- This motivated Einstein's equivalence principle

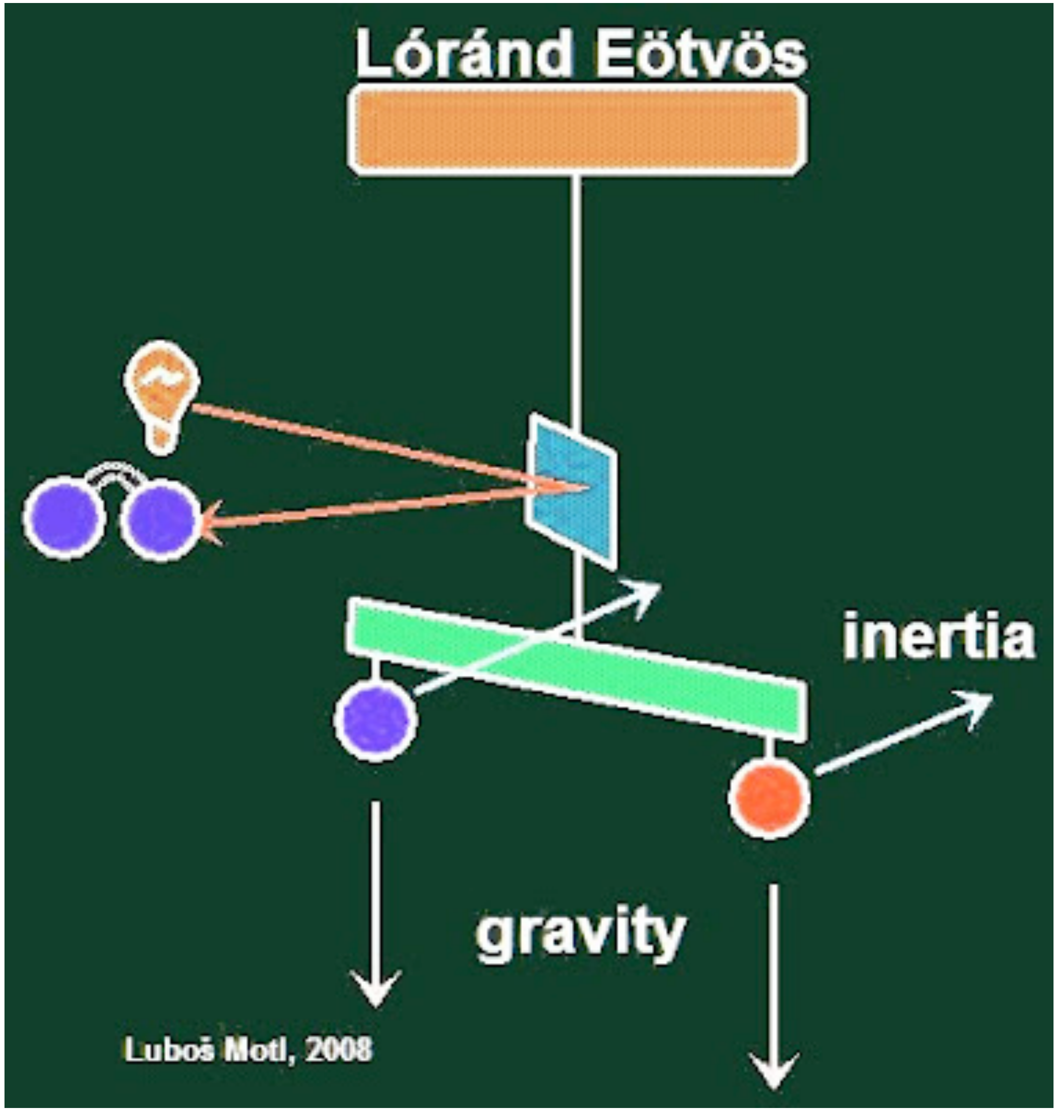
  "The laws of motion in a freely-falling, non-rotating lab occupying a small region of space-time are those of Special Relativity"

In the strong form, this applies to all physical laws

- A key test of this is to attempt to balance a gravitational and inertial force – if there is an inbalance between the masses, a resultant force will create motion

# Mass Equivalence





# Mass Equivalence

- Original Eotvos experiment (1922) $(m_I/m_g)-1 < 5\text{x}10^{-9}$
- Dicke,Roll & Krotkov (1964) $(m_I/m_g)-1 < 3\text{x}10^{-11}$
- Shapiro et al (1976) $(m_I/m_g)-1 < 1\text{x}10^{-12}$
- Adelberger et al (1990) $(m_I/m_g)-1 < 1\text{x}10^{-12}$
- Baessler et al (1999) $(m_I/m_g)-1 < 5\text{x}10^{-13}$

PRL **100**, 041101 (2008) PHYSICAL REVIEW LETTERS week ending 1 FEBRUARY 2008

**Test of the Equivalence Principle Using a Rotating Torsion Balance**

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(Received 4 October 2007; revised manuscript received 3 December 2007; published 28 January 2008)

We used a continuously rotating torsion balance instrument to measure the acceleration difference of beryllium and titanium test bodies towards sources at a variety of distances. Our result $\Delta a_{N,\text{Be-Ti}} = (0.6 \pm 3.1) \times 10^{-15}$ m/s$^2$ improves limits on equivalence-principle violations with ranges from 1 m to $\infty$ by an order of magnitude. The Eötvös parameter is $\eta_{\text{Earth,Be-Ti}} = (0.3 \pm 1.8) \times 10^{-13}$. By analyzing our data for accelerations towards the center of the Milky Way we find equal attractions of Be and Ti towards galactic dark matter, yielding $\eta_{\text{DM,Be-Ti}} = (-4 \pm 7) \times 10^{-5}$. Space-fixed differential accelerations in any direction are limited to less than $8.8 \times 10^{-15}$ m/s$^2$ with 95% confidence.

# Lunar Ranging

The Moon is in Earth Orbit
Precision measurements of the orbit, its precession and shape test GR in the weak field
Done by Lunar Ranging

VOLUME 36 15 MARCH 1976 NUMBER 11

## New Test of the Equivalence Principle from Lunar Laser Ranging*

J. G. Williams, R. H. Dicke, P. L. Bender, C. O. Alley, W. E. Carter, D. G. Currie, D. H. Eckhardt, J. E. Faller, W. M. Kaula, J. D. Mulholland, H. H. Plotkin, S. K. Poultney, P. J. Shelus, E. C. Silverberg, W. S. Sinclair, M. A. Slade, and D. T. Wilkinson

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(Received 8 December 1975)

An analysis of six years of lunar-laser-ranging data gives a zero amplitude for the Nordtvedt term in the Earth-Moon distance yielding the Nordtvedt parameter $\eta = 0.00 \pm 0.03$. Thus, Earth's gravitational self-energy contributes equally, $\pm 3\%$, to its inertial mass and passive gravitational mass. At the 70% confidence level this result is only consistent with the Brans-Dicke theory for $\omega > 29$. We obtain $|\beta - 1| \lesssim 0.02$ to 0.05 for five-parameter parametrized post-Newtonian theories of gravitation with energy-momentum conservation, or $|\beta - 1| \lesssim 0.01$ if only $\beta$ and $\gamma$ are considered.

# Lunar Ranging

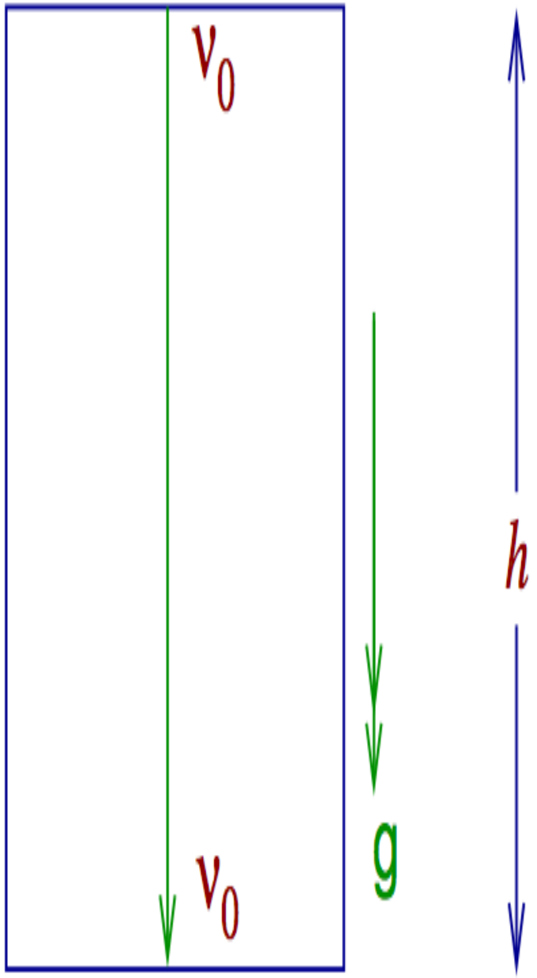
Experiments started by Russian landers and Apollo astronauts

Now over 40 years of data.

Tests: equivalence principle, orbits, time dilation

[BBC Horizon Clip]

# Time Dilation in the weak field




Consider a falling laboratory, dropped at the same moment light leaves the ceiling.

Light takes time $t \approx h/c$ to reach floor by which time lab is moving downwards with speed $v = gh/c$

An observer in the lab sees no change in the light frequency. An observer at a fixed point outside the lab, sees frequency

$$\nu_1 \approx \nu_0 \left(1 + \frac{v}{c}\right) = \nu_0 \left(1 + \frac{gh}{c^2}\right) = \nu_0 \left(1 + \frac{\phi}{c^2}\right)$$

=> Clocks run faster at ceiling than floor

See Reading on website for Pound and Rebka experiment

# Time Dilation on/near Earth

Q7.6d

The GPS satellites provide a famous practical example of GR. The GPS satellites orbit at a radius of 26,600 km around Earth which has mass $5.98 \times 10^{24}$ kg and radius 6370 km. Calculate the rate at which the GPS satellites gain or lose compared to clocks on Earth, and hence, given that they provide positional information by timing, calculate the positional error that could result after one day of ignoring relativistic effects.

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*A subtlety here is that we need to account for the relativistic effects on Earth as well as the satellite. The difference between the rates at which clocks run on Earth cf the satellites (scaled by an observer at infinity)* $= \alpha_E - \alpha_{GPS}$, *which, ignoring Earth's rotation, evaluates to*

$$= \frac{3 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{2 \times (3.00 \times 10^8)^2 \times 2.66 \times 10^7} - \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(3.00 \times 10^8)^2 \times 6.37 \times 10^6} = 2.5 \times 10^{-10} = -4.46 \times 10^{-10}.$$

*The negative sign indicates that the Earth clocks runs slower than the satellites. In one day (86400 sec) this amounts to a drift of* 38.5 *microseconds, the equivalent of* 11 km *positional error!*

Also tested by experiments like Gravity Probe A

# Shapiro Delay

- For coordinate time in the Schwarzschild metric, for photons following radial paths:

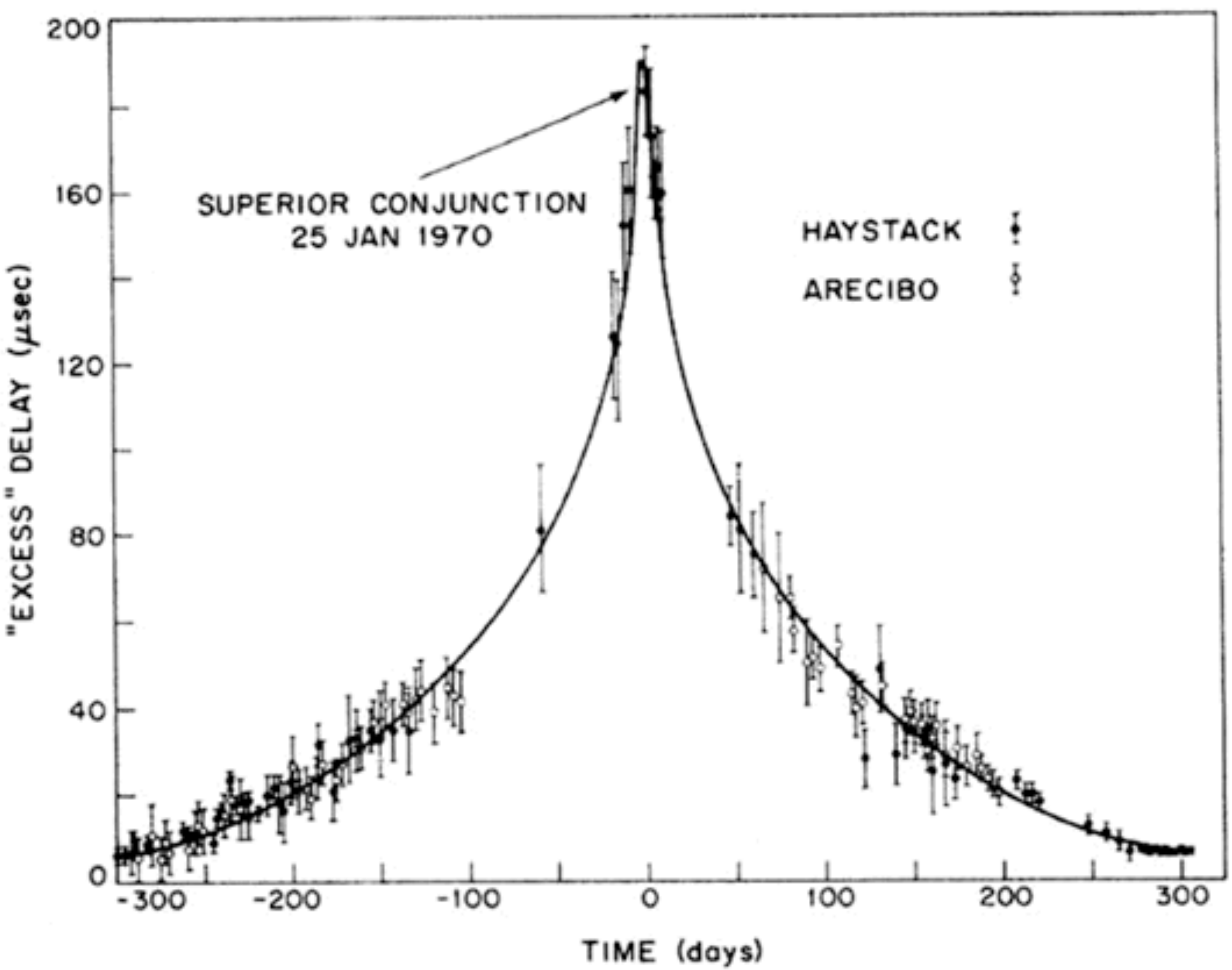
$$c\,dt = \frac{dr}{1 - 2GM/c^2 r}.$$

$$\begin{aligned} c(t_2 - t_1) &= \int_{r_1}^{r_2} \frac{dr}{1 - 2GM/c^2 r} \\ &= (r_2 - r_1) + \frac{2GM}{c^2} \ln \frac{r_2 - 2GM/c^2}{r_1 - 2GM/c^2} \end{aligned}$$

This last term is known as the 'Shapiro Delay' and is another key test of GR. Shapiro bounced radar off Venus and Mercury, testing the effects of the Solar Potential

See Q6.4

# Shapiro Effect

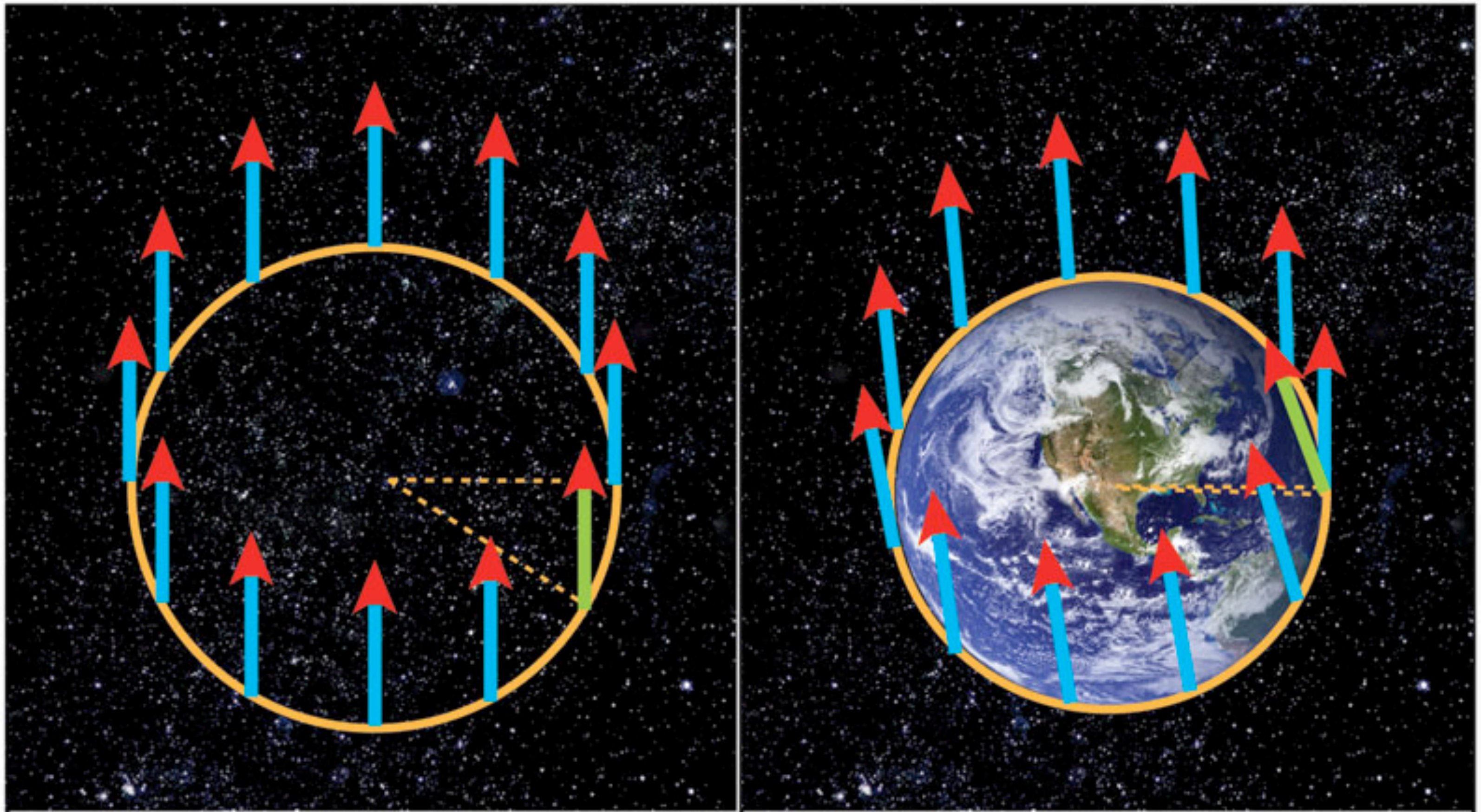


# Gravity Probe B

- Flew in 2004
- NASA mission
- Tested:
  - Frame Dragging (effect due to rotation)
  - Geodetic effect (effect due to curvature)

See einstein.stanford.edu

# Gravity Probe B



A circle with Earth's equatorial diameter (~7,926 mi) in empty space has a circumference of $\pi D$ (~24,901 mi). A gyroscope following this circular path in empty space will always point in the same direction, as indicated by the arrows above.

Earth's mass warps spacetime inside the circle into a cone, formed by removing a pie-shaped wedge (dotted lines). This reduces the circle's circumference by 1.1 inches. A gyroscope will now change its orientation while tracing the conical path, as shown in the drawing above.

# Precession in Schwarzschild Geometry

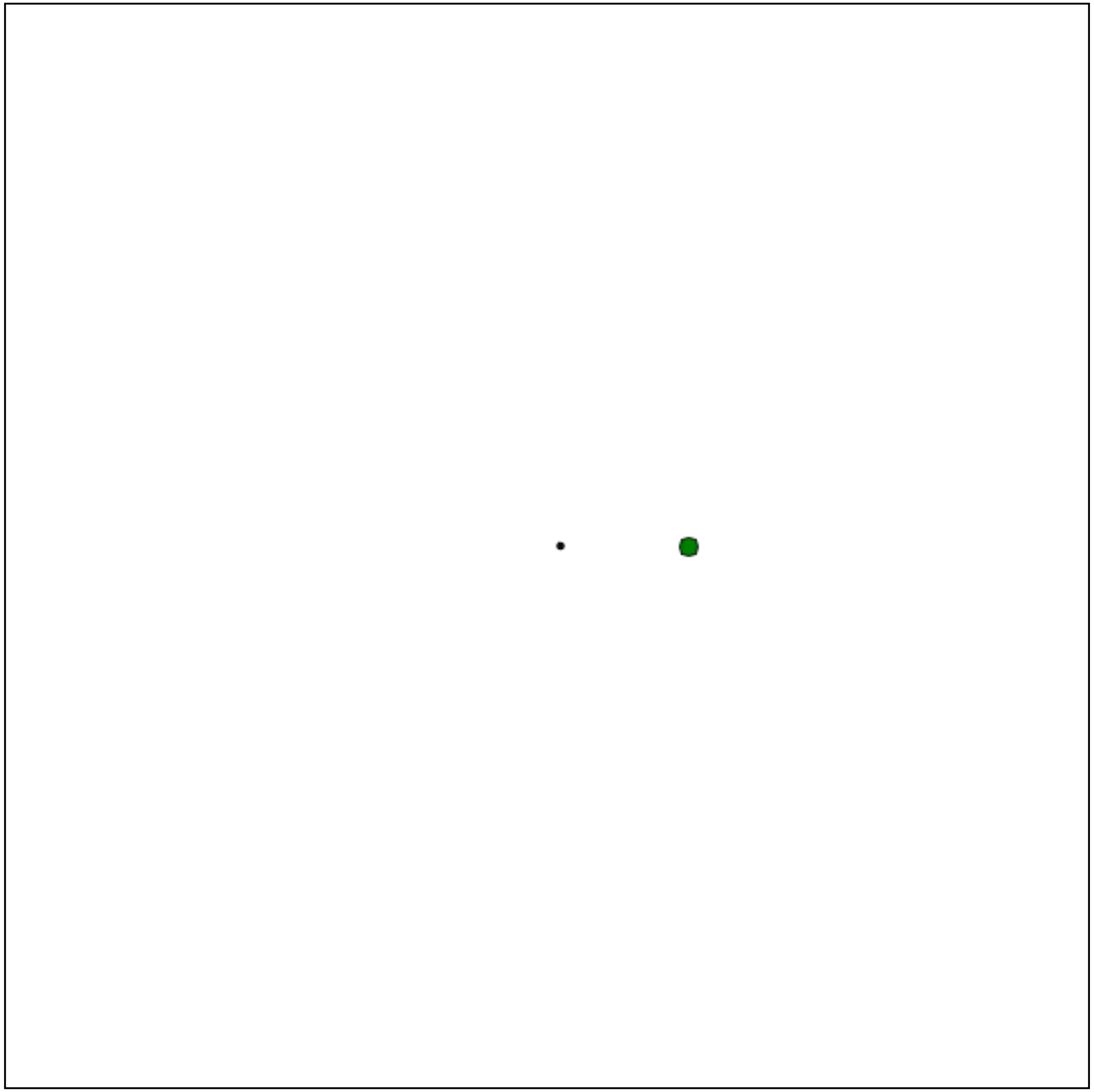
$$\begin{aligned}\Delta\phi &= 2\pi\left[\frac{1}{r^2}\left(\frac{\mu c^2 r^2}{r-3\mu}\right)^{1/2}\left(\frac{r-3\mu}{r-6\mu}\right)^{1/2}\left(\frac{r^3}{\mu c^2}\right)^{1/2}-1\right],\\ &= 2\pi\left[\left(\frac{r}{r-6\mu}\right)^{1/2}-1\right]\ \text{rads/orbit}\end{aligned}$$

If $r \gg \mu$ this can be approximated as $\delta\phi \approx 6\pi\mu/r$ rads/orbit, or

$$\delta\phi \approx \frac{6\pi GM}{c^2 r}\ \text{rads/orbit.}$$

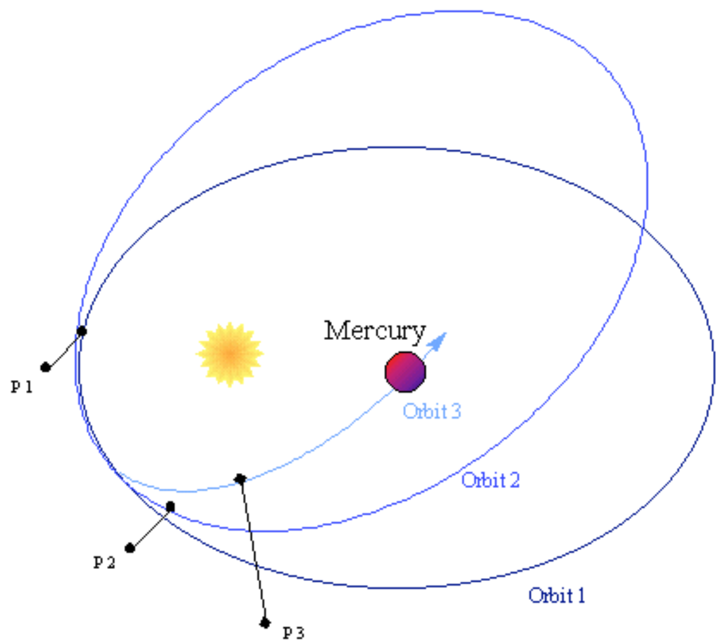
The precession is in the direction of the orbit (prograde).

# Precession in Schwarzschild Metric



# Precession of the Perihelion of Mercury

- Mercury's perihelion precesses 5600 arcsec/ century
- Of that 42.98+-0.04 arcsec/ century was considered anomalous
- GR predicts 43 arcsec/ century

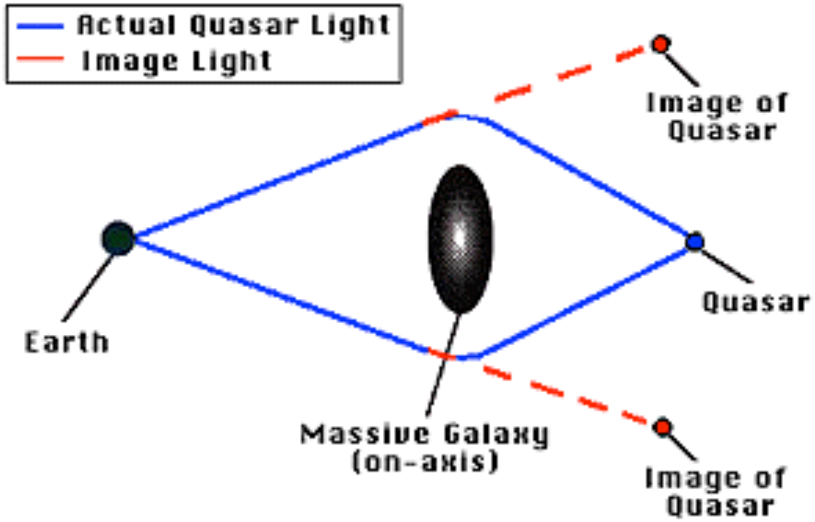



- This was the only experimental evidence against Newtonian Dynamics when Einstein formulated GR.
- Same effect seen in binary pulsars which can precess $17^o$/yr

# Deflection of Light

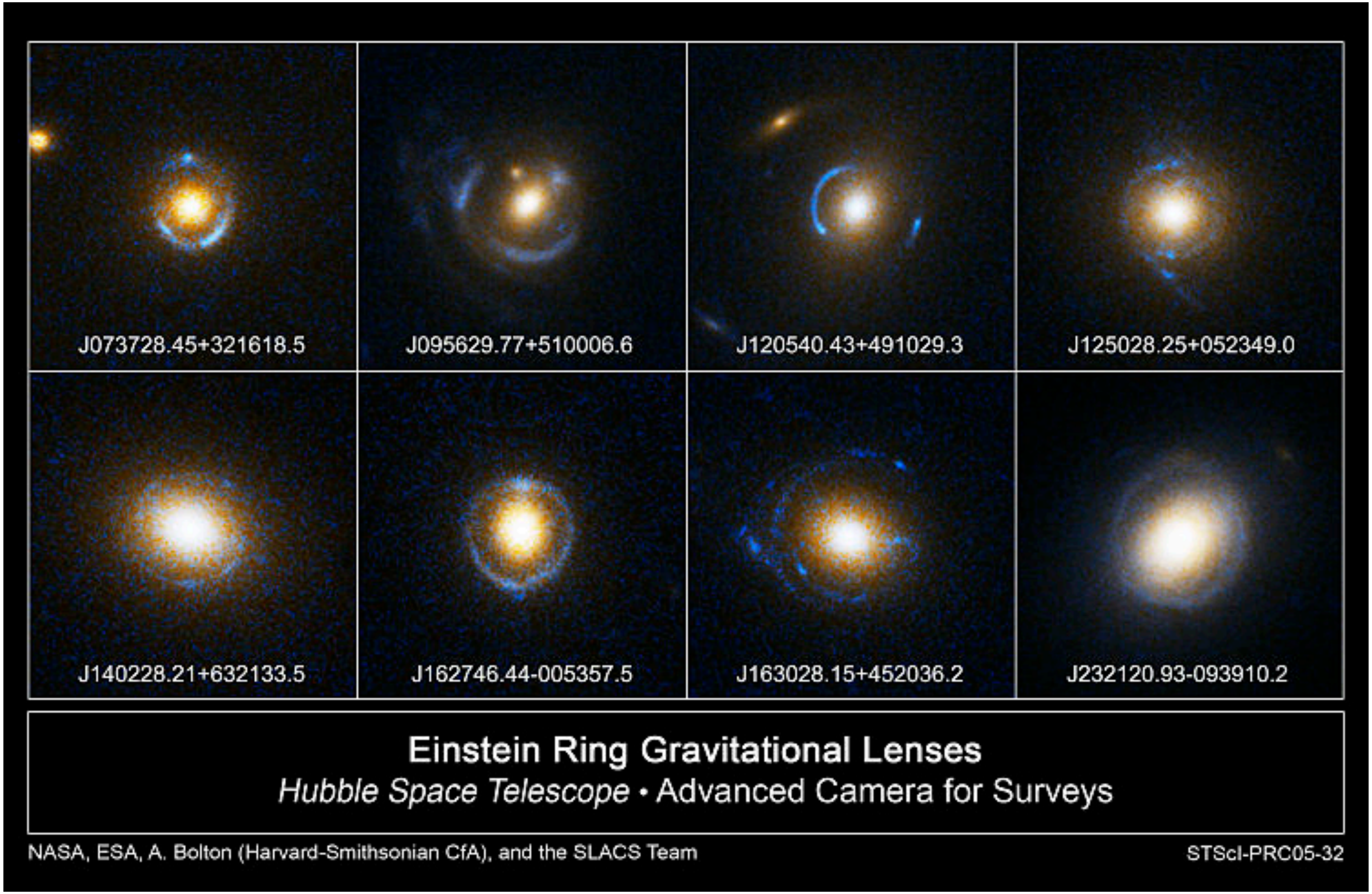
- Light passing through a Schwarzschild metric is deflected through an angle

$$\Delta\phi = 4\mu a = \frac{4GM}{c^2 r_0}.$$



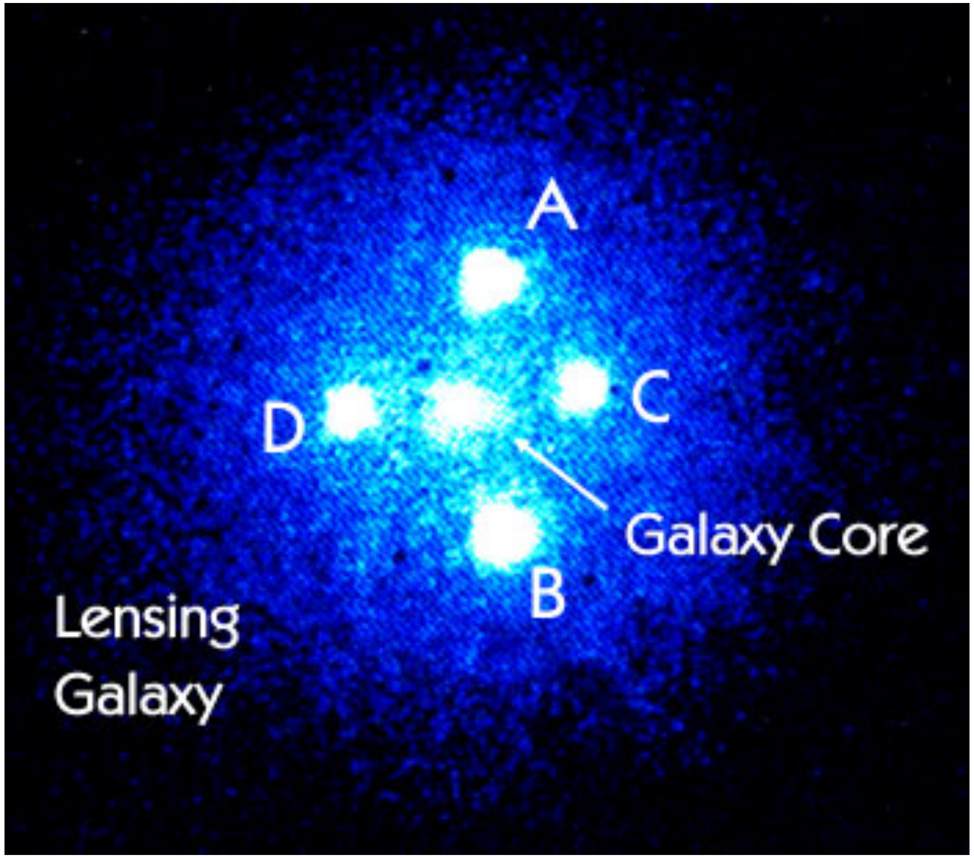

- If $\Delta\varphi$ is measured, and other parameters are known, then this can be used to test GR. Otherwise it can be used to measure properties such as mass of the system.

# Einstein Rings



# Einstein Cross

- The **Einstein Cross** or *Q2237+030* is a gravitationally lensed quasar that sits directly behind ZW 2237+030



# Strong Lensing Examples

- Interpreting lensing can get complicated

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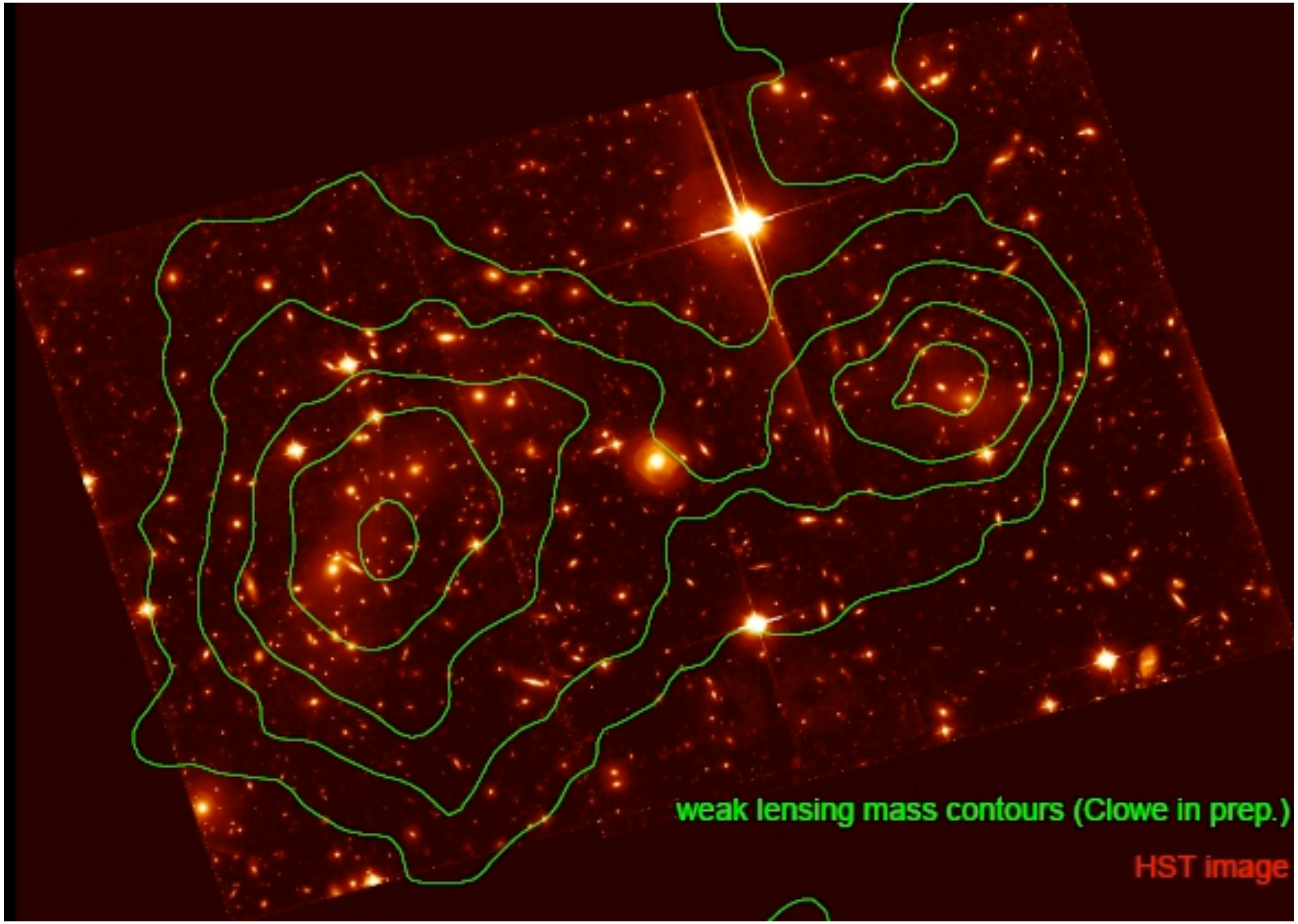
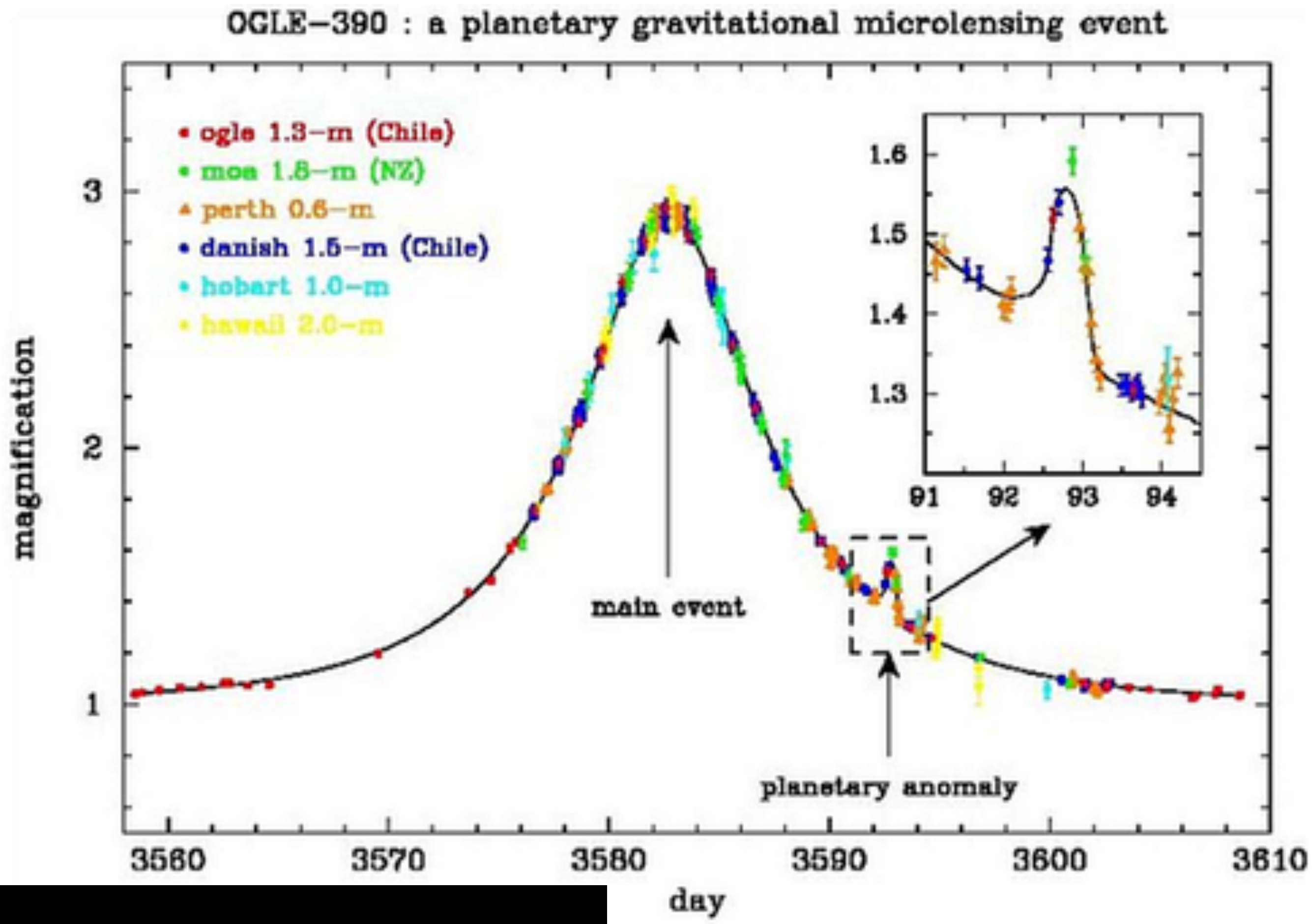
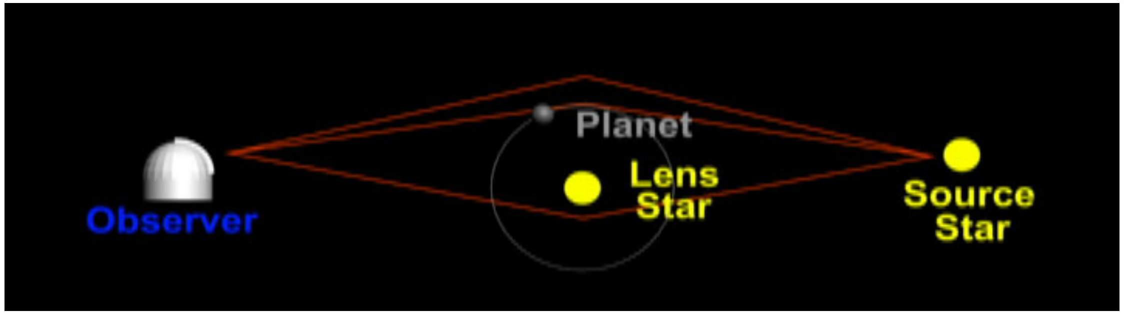
- Interpreting lensing can get complicated



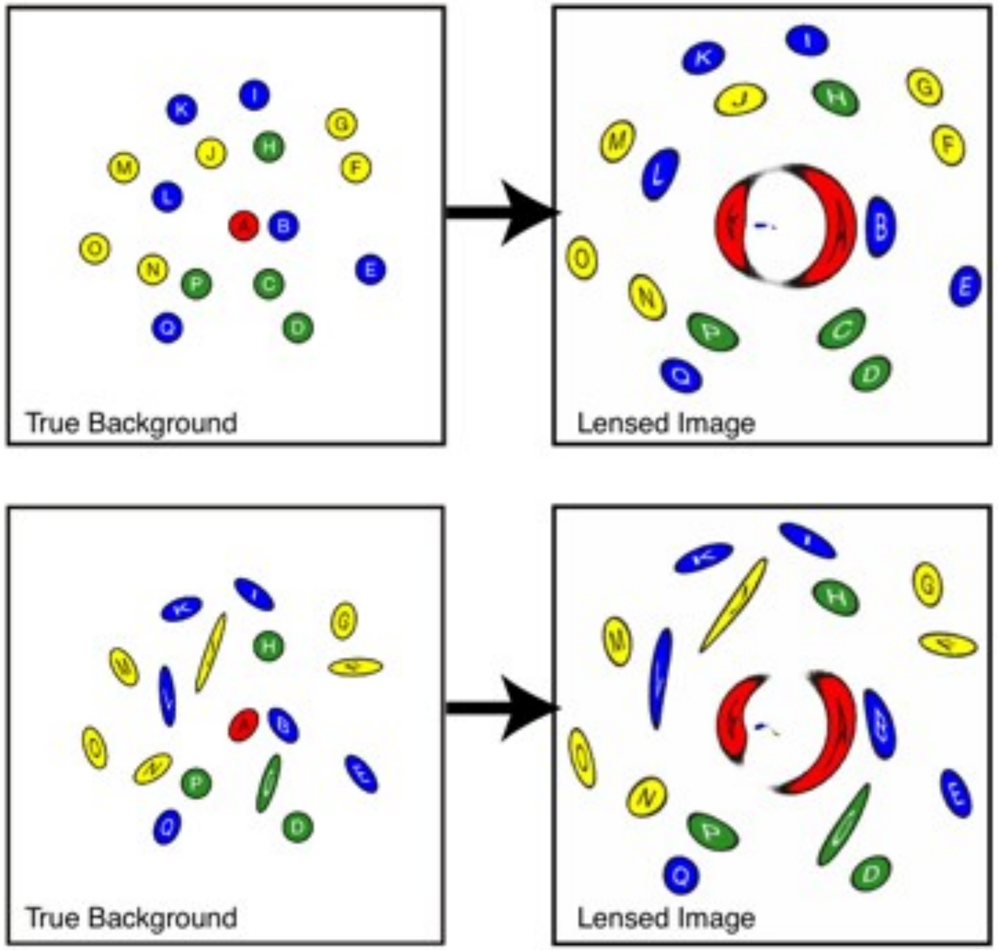
# Microlensing



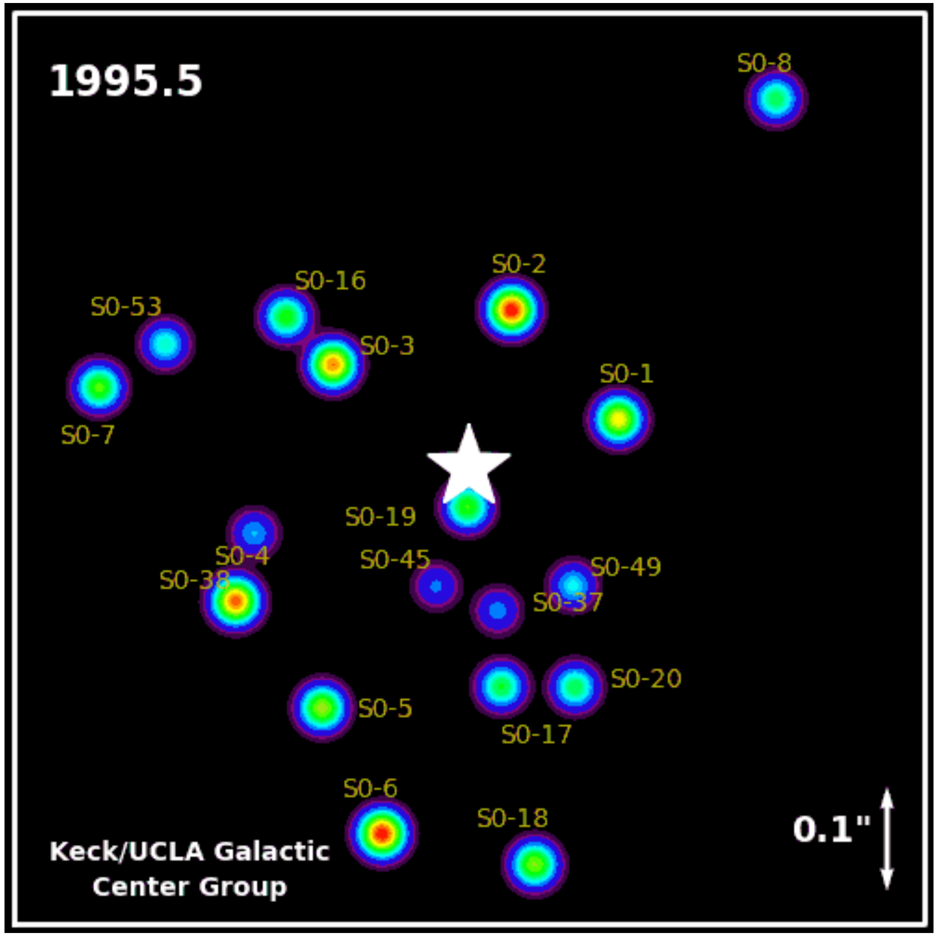
# Weak Lensing

- For sources a long way off-axis, the distortion might not be visible.
- By measuring the shape of hundreds of galaxies, can look for very slight distortions



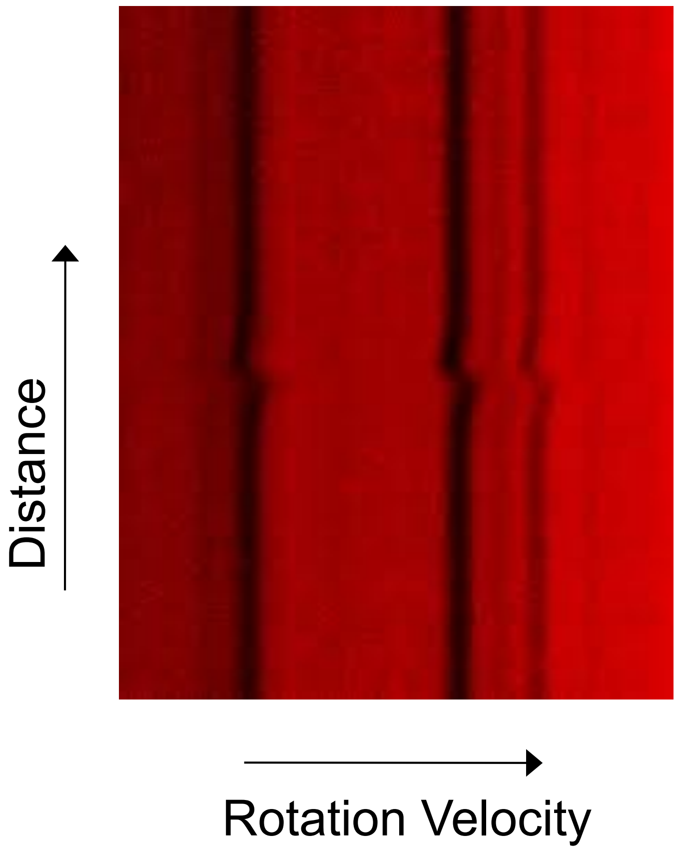
# Black Holes and Accretion

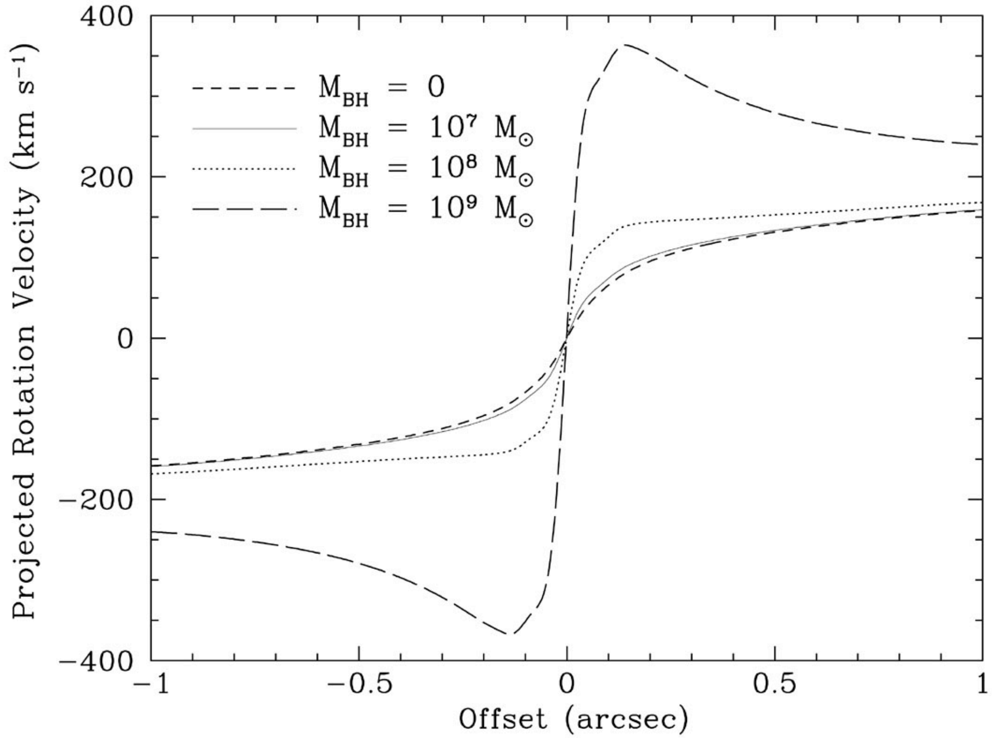
- Orbits of stars around the centre of our galaxy provide strong evidence for the existence of a black hole



# Black Holes and Accretion

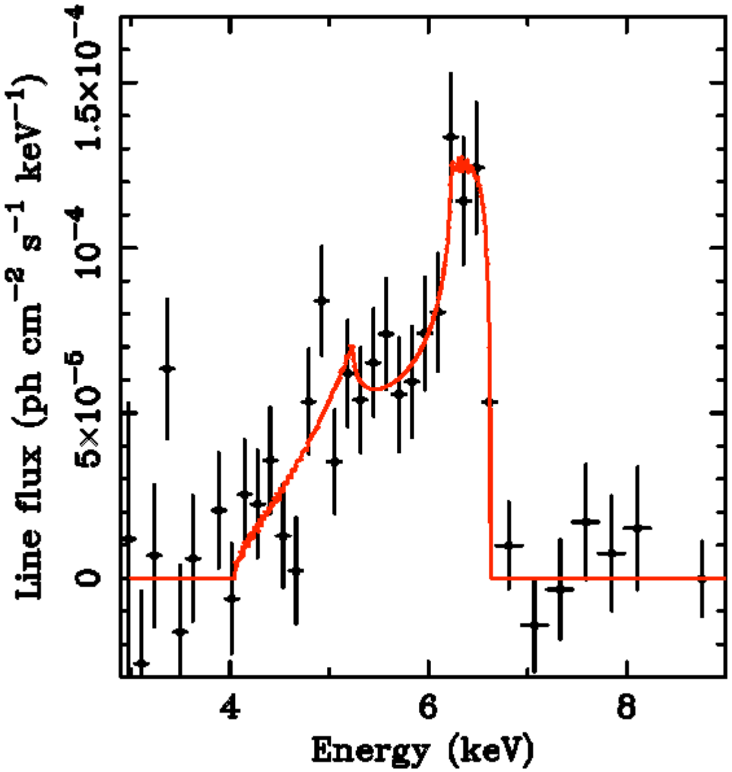
- Orbits of stars in other galaxies also provide evidence for Black Holes (indirect evidence for GR)



# Black Holes and Accretion

- Light does not escape from a black hole (i.e. within $R_s=2GM/c^2$ in the Schwarzschild metric)
- Nonetheless we observe Black hole-containing systems as some of the luminous in the universe




Material falling onto the BH forms an accretion disk – this can release ~4% of the gravitational potential

The disk can be subject to GR effects

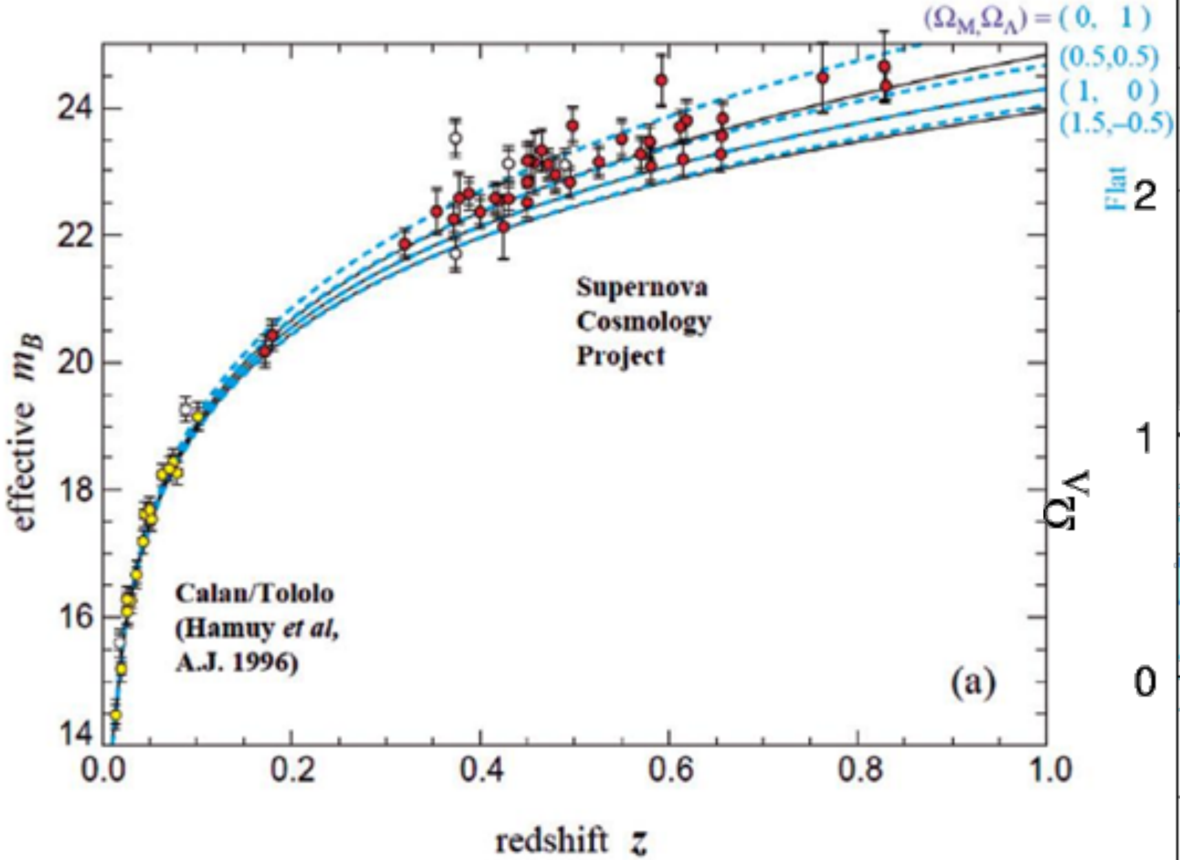
e.g. MCG-6-30-15 (Tanaka et al 1995), broad Iron K line fits a disk extending from 3-10 Rs at 30$^o$ to line of sight

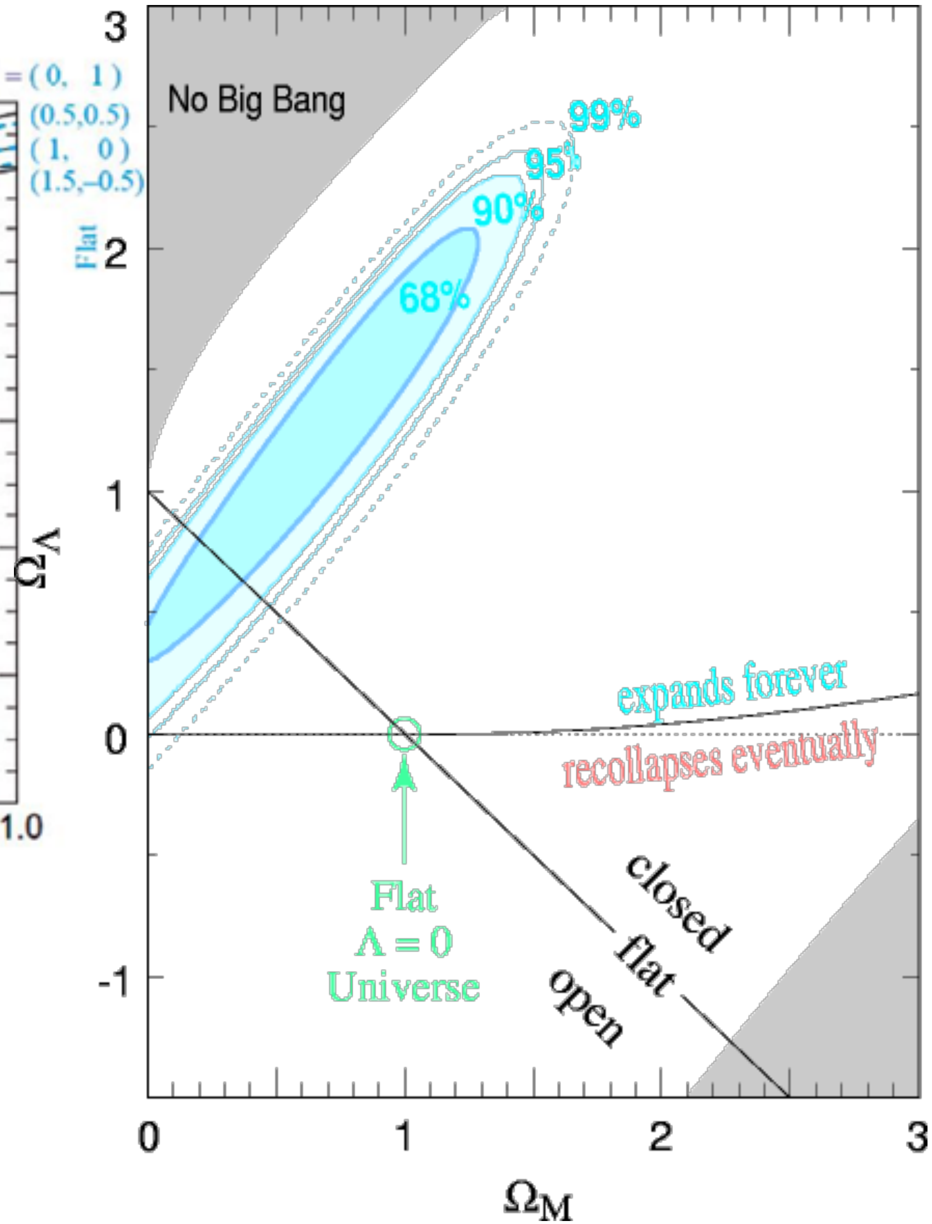
# Large Scale Tests in Cosmology

- Expansion of the Universe (inevitable consequence of Einstein's field equations in the FRW metric)
- Weak Lensing on Large Scales (deflection of light, integrated from many weak sources)
- Ripples in the CMB caused by Early Universe perturbations (most of which are GR sensitive)

- Future instruments like LSST and DES will probe these effects further

# Distance Measures in Cosmology

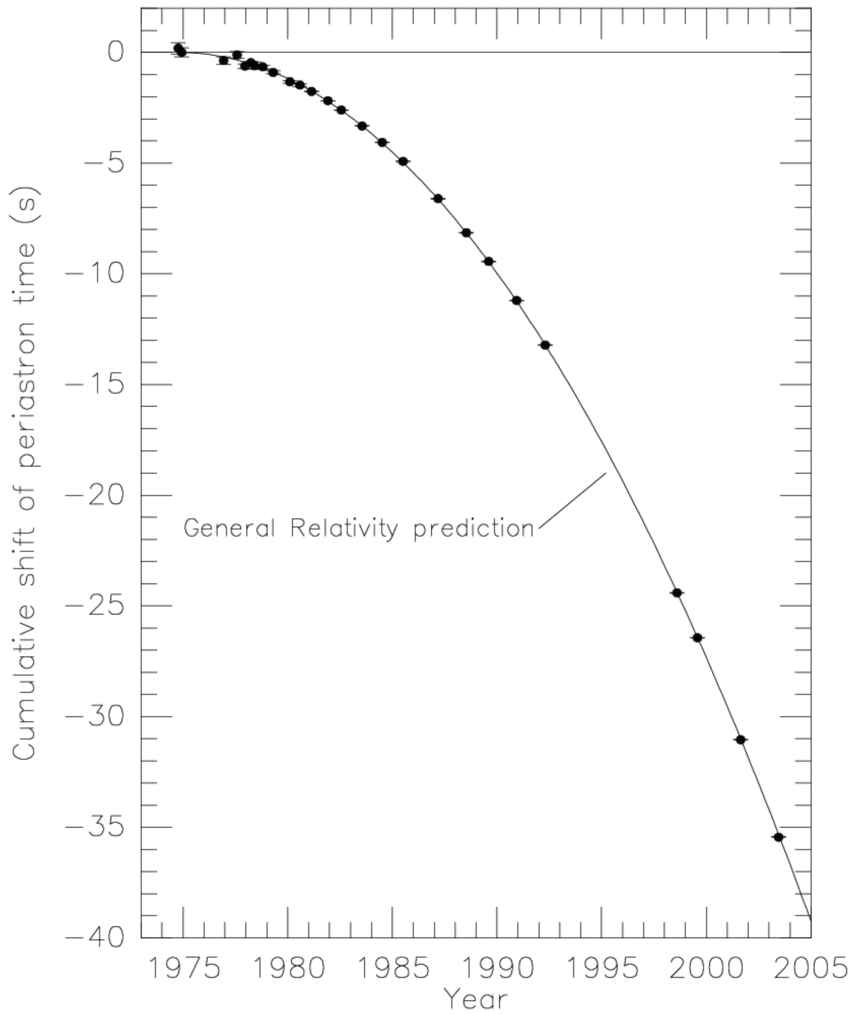


# Binary Pulsars

- Strong evidence for GR comes from observed spin-down of binary pulsars, compared to the predictions for energy loss due to gravitational waves

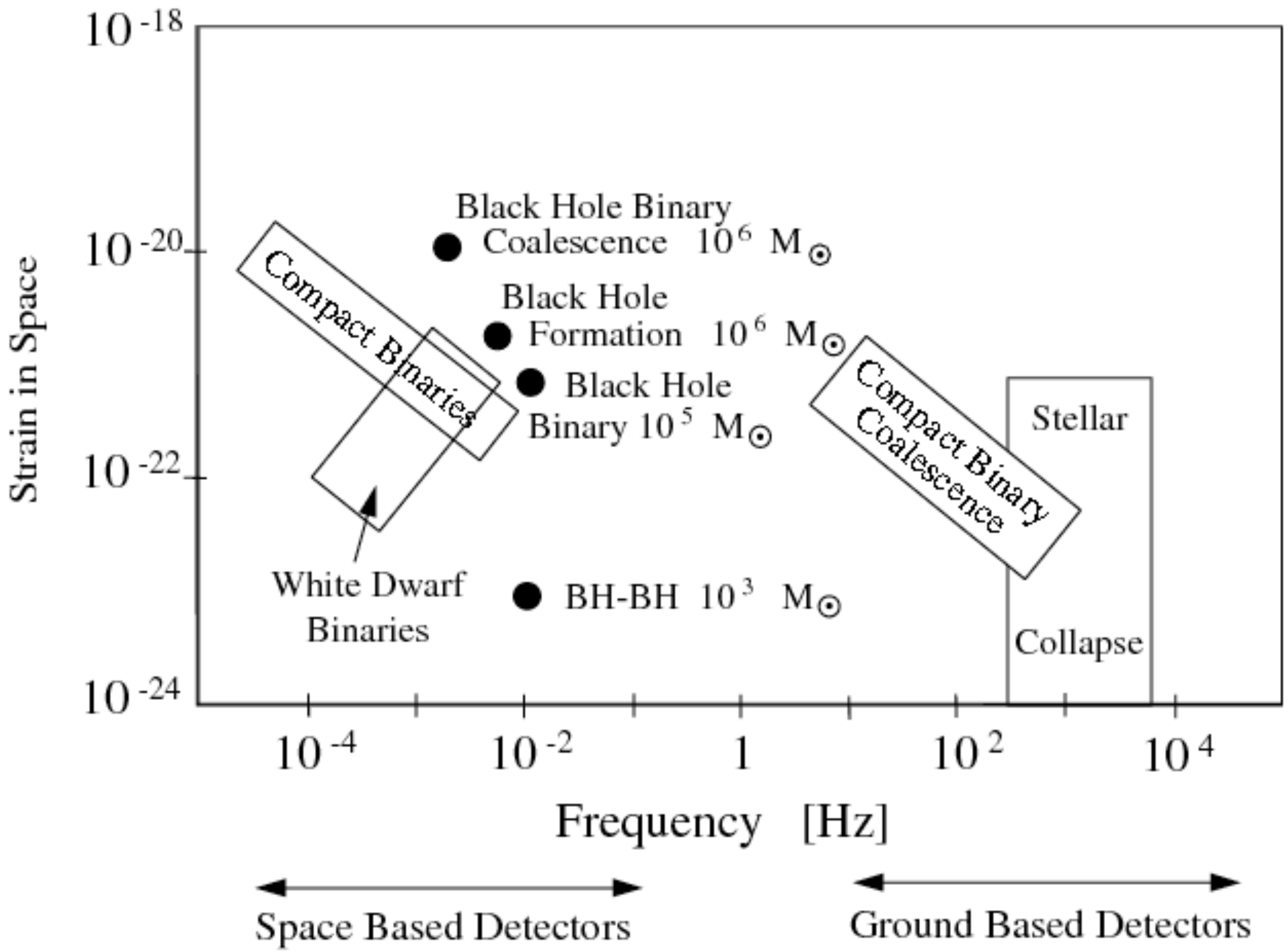
- This is a strong field test of GR




e.g. PSR1916+013 (Nobel Prize 1993, Hulse and Taylor)

# Gravitational Waves

- The strength of the gravitational perturbation is usually given in the form of a *strain amplitude, h*



# Gravitational Waves

- Q10.2 – Can we use a local source to calibrate these detectors?

Using the formula from lectures for the amplitude of gravitational waves produced by a variable mass quadrupole at distance $r$

$$\bar{h}^{ij} = -\frac{2G}{c^4 r}\frac{d^2 I^{ij}}{dt^2},$$

where

$$I^{ij} = \int \rho x^i x^j \, dV,$$

discuss the feasibility of setting up a calibration source for the current laser interferometers consisting of two equal masses rotated around a vertical axis. Assume a sensitivity of $h \sim 10^{-21}$ at frequencies $\sim 100$ Hz.

# Gravitational Waves

- Q10.2 – Can we use a local source to calibrate these detectors?

*Assuming that each mass $m$ is a distance $a$ from the axis then their $x$ coordinates are given by*

$$x = \pm a \cos \omega t.$$

*Assuming point masses, the quadrupole integral reduces to a sum over each mass:*

$$I^{xx} = m(a\cos\omega t)^2 + m(a\cos\omega t)^2 = ma^2(1 + \cos 2\omega t).$$

*Differentiating twice,*

$$\ddot{I}^{xx} = -4ma^2\omega^2 \cos 2\omega t.$$

*Therefore the $\bar{h}^{xx}$ component oscillates with amplitude*

$$8Gma^2\omega^2$$

# Gravitational Waves

- Q10.2 – Can we use a local source to calibrate these detectors?

*For a given target $h$ this defines the parameters required for the calibration source. The interferometers are several kilometres in length so let's take $r = 100\,\mathrm{km}$ to make sure the waves produced by the calibration source are roughly planar. Therefore*

$$ma^2\omega^2 = \frac{c^4 hr}{8G} \sim 10^{26}\,\mathrm{J}.$$

*This corresponds to $\sim 10^{16}\,\mathrm{J}$ per person on the planet, equivalent to 300 $MW$ per person for a year. This is not going to happen!*

So the answer is no… but this also demonstrates just how much energy systems that *do* generate gravitational wave require – gravitational potential is one of the most powerful energy sources in the Universe

# Final Summary

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# Final Summary

- GR is an elegant and (relatively) simple solution to the problem of ‘action at a distance’ in Newtonian gravity
- Einstein’s field equations can be applied in a variety of metrics which trace the underlying geometry and predict particle and photon geodesics
- GR has observable consequences which have been tested to a high degree of precision
- Any rival theory must explain not just one of these observations, but all of them – so far no theory exists that competes with GR on precision and simplicity.