PHY 332

Atomic and Laser Physics

Part II: Laser Physics

9. Introduction to lasers

Overview

LASER = "light amplification by stimulated emission of radiation"

This is a bit of a misnomer. A laser is actually an *oscillator* rather than a simple amplifier. The difference is that an oscillator has positive feedback in addition to the amplifier. "Light" is understood in a general sense of electromagnetic radiation with wavelength around 1 micron. Thus one can have infrared, visible or ultraviolet lasers.

Milestones in the history of lasers:

1917	Einstein's treatment of stimulated emission.
1951	Development of the maser by C.H. Townes. The maser is basically the same idea as the laser, only it works at microwave frequencies.
1958	Proposal by C.H. Townes and A.L. Schawlow that the maser concept could be extended to optical frequencies.
1960	T.H. Maiman at Hughes Laboratories reports the first laser: the pulsed ruby laser.
1961	The first continuous wave laser is reported (the helium neon laser).
1964	Nicolay Basov, Charlie Townes and Aleksandr Prokhorov get the Nobel prize for “fundamental work in the field of quantum electronics, which has led to the construction of oscillators and amplifiers based on the maser-laser principle.”
1981	Art Schalow and Nicolaas Bloembergen get the Nobel Prize for “their contribution to the development of laser spectroscopy.”
1997	Steven Chu, Claude Cohen-Tannoudji and William D. Phillips get the Nobel Prize for the “development of methods to cool and trap atoms with laser light.”
2005	John Hall and Theodor Hänsch receive the Nobel Prize for “their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique”.

Types of laser

Lasers come in many shapes and sizes. They are classified by various criteria:
- Gain medium is solid, liquid or gas
- Wavelength is in the infrared, visible or ultraviolet spectral region
- Mode of operation is continuous or pulsed
- Wavelength is fixed or tuneable.

The present state of the art includes:
- Peak powers > 10^{12} W;
- Pulses shorter than 10^{-15} s;
- Cheap, efficient diode lasers available at blue (400 nm), red (620–670nm), and near-infrared wavelengths (700–1600 nm);
- Other readily-available fixed-wavelengths include:
  - *Infrared*: CO_2 (10.6 μm), erbium (1.55 μm), Nd:YAG (1.064 μm), Nd:glass (1.054 μm);
  - *Visible*: ruby (693nm), Kr+ (676, 647 nm), HeNe (633 nm), Cu (578 nm), doubled Nd:YAG (532 nm), Ar+ (514, 488 nm), HeCd (442 nm);
  - *Ultraviolet*: Ar+ (364, 351 nm), tripled Nd:YAG (355 nm), N_2 (337 nm) HeCd (325 nm), quadrupled Nd:YAG (266 nm), excimer (308, 248, 193, 150 nm) ;

- Tuneable lasers:
  - Dye (typical tuning range ~ 100 nm, dyes available from UV to near infrared);
  - Ti: sapphire (700–1000 nm, doubled: 350–500nm);
  - Free electron (far infrared to ultraviolet).

Properties of Laser light

1. MONOCHROMATIC.

The emission of the laser generally corresponds to just one of the atomic transitions of the gain medium, in contrast to discharge lamps, which emit on all the transitions. The spectral line width can be much smaller than that of the atomic transition. This is because the emission is affected by the optical cavity. In certain cases, the laser can be made to operate on just one of the modes of the cavity. Since the *Q* of the cavity is generally rather large, the mode is usually much narrower than the atomic transition, and the spectral line width is orders of magnitude smaller than the atomic transition. This is particularly useful for high resolution spectroscopy and applications such as interferometry and holography that require high coherence (see below).

2. COHERENCE.

In discussing the coherence of an optical beam, we must distinguish between spatial and temporal coherence. Laser beams have a high degree of both.

Spatial coherence refers to whether there are irregularities in the optical phase in a cross-sectional slice of the beam.

Temporal coherence refers to the time duration over which the phase of the beam is well defined. In general, the temporal *coherence time* t_c is given by the reciprocal of the spectral line width $\Delta\nu$. Thus the *coherence length* l_c is given by:

$$l_c = c\, t_c = c\frac{1}{\Delta\nu} \qquad (9.1)$$

Typical values of the coherence length for a number of light sources are given in Table 9.1. The figures explain why it is much easier to do interference experiments with a laser than with a discharge lamp. If the path difference exceeds l_c you will not get interference fringes, because the light is incoherent. In the case of the single mode HeNe laser, you can set up an interferometer in which the path lengths differ by 300 m, and you will still observe fringes. The long coherence length of laser light is useful in holography and interferometry.

SOURCE	Spectral line width $\Delta\nu$ (Hz)	Coherence time t_c (s)	Coherence length l_c
Sodium discharge lamp (D-lines at 589nm)	5×10^{11}	2×10^{-12}	0.6 mm
Multi-mode HeNe laser 632.8nm line	1.5×10^{9}	6×10^{-10}	20 cm
Single-mode HeNe laser 632.8nm line	1×10^{6}	1×10^{-6}	300 m

TABLE 9.1: Coherence length of several light sources.

3. DIRECTIONALITY.

This is perhaps the most obvious aspect of a laser beam: the light comes out as a highly directional beam. This contrasts with light bulbs and discharge lamps, in which the light is emitted in all directions. The directionality is a consequence of the cavity.

4. BRIGHTNESS.

The brightness of lasers arises from two factors. First of all, the fact that the light is emitted in a well-defined beam means that the power per unit area is very high, even though the total amount of power can be rather low. Then we must consider that all the energy is concentrated within the narrow spectrum of the active atomic transition. This means that the spectral brightness (i.e. the intensity in the beam divided by the width of the emission line) is even higher in comparison with a white light source like a light bulb. For example, the spectral brightness of a 1 mW laser beam could easily be millions of time greater than that of a 100 W light bulb.

Properties 1-4 are common to all lasers. There is a fifth property which is found in only some:

5. ULTRASHORT PULSE GENERATION.

Lasers can be made to operate continuously or in pulses. The time duration of the pulses t_p is linked to the spectral band width of the laser light $\Delta\nu$ by the "uncertainty" product $\Delta t\,\Delta\nu \sim 1$:

$$t_p \gtrsim \frac{1}{\Delta\nu} \ . \tag{9.2}$$

This follows from taking the Fourier transform of a pulse of duration t_p. As an example, the bandwidth of the 632.8 nm line in the HeNe laser is 1.5 GHz (see Table 9.1 above), so that the shortest pulses that a HeNe can produce would be 0.67 ns long. This is not particularly short by modern standards. Dye lasers typically have gain bandwidths greater than 10^{13} Hz, and can be used to generate pulses shorter than 100 fs (1 fs = 10^{-15} s). This is achieved by a technique called "mode-locking". The present world record for a light pulse is less than 1 fs. These short pulsed lasers are very useful for studying fast processes in physics, chemistry and biology, and are also being developed for high data rate transmission using optical fibre networks.

Essential operating principles of a laser

light amplification + positive optical feedback

Light amplification is achieved by *stimulated emission*. (See below.) Ordinary optical materials do not amplify light. Instead, they tend to absorb or scatter the light, so that the light intensity out of the medium is less than the intensity that went in. To get amplification you have to drive the material into a non-equilibrium state by pumping energy into it. The amplification of the medium is determined by the *gain coefficient* γ, which is defined by the following equation:

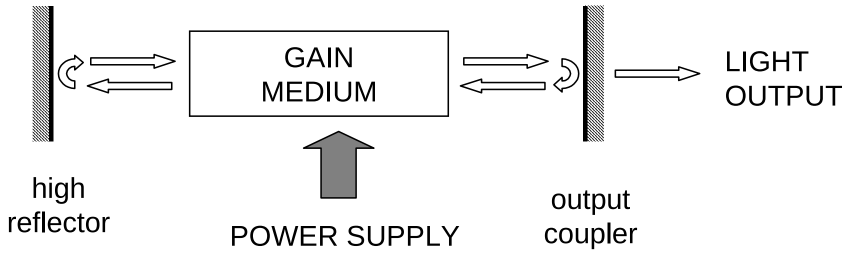
$$\begin{aligned} I(x+dx) &= I(x) + \gamma\, I(x)\, dx \\ &= I(x) + dI \end{aligned} \tag{9.3}$$

where $I(x)$ represents the intensity at a point x within the gain medium. The differential equation can be solved as follows:

$$\begin{aligned} dI &= \gamma\, I\, dx \\ \frac{\mathrm{d}I}{\mathrm{d}x} &= \gamma\, I \\ \therefore I(x) &= I(0)\ \mathrm{e}^{\gamma x} \end{aligned} \tag{9.4}$$

Thus the intensity grows exponentially within the gain medium.

Positive optical feedback is achieved by inserting the amplifying medium inside a resonant cavity:



Light in the cavity passes through the gain medium and is amplified. It then bounces off the end mirrors and passes through the gain medium again, getting amplified further. This process repeats itself until a stable equilibrium condition is achieved when the total round trip gain balances all the losses in the cavity. Under these conditions the laser will oscillate. The condition for oscillation is thus:

round-trip gain = round-trip loss

The losses in the cavity fall into two categories: useful, and useless. The useful loss comes from the output coupling. One of the mirrors (called the "output coupler") has reflectivity less than unity, and allows some of the light oscillating around the cavity to be transmitted as the output of the laser. The value of the transmission is chosen to maximise the output power. If the transmission is too low, very little of the light inside the cavity can escape, and thus we get very little output power. On the other hand, if the transmission is too high, there may not be enough gain to sustain oscillation, and there would be no output power. The optimum value is somewhere between these two extremes. Useless losses arise from absorption in the optical components (including the laser medium), scattering, and the imperfect reflectivity of the other mirror (the "high reflector"). Taking into account the fact that the light passes twice through the gain medium during a round trip, the condition for oscillation in a laser with small gain and losses can be written:

$$2\gamma l = (1 - R_{OC}) + (1 - R_{HR}) + \text{ scattering losses } + \text{ absorption losses }, \qquad (9.5)$$

where l is the length of the gain medium, R_{OC} is the reflectivity of the output coupler and R_{HR} is the reflectivity of the high reflector. When the gain and losses are large (i.e. >10%), the more general version of the oscillation condition must be used:

$$e^{2\gamma l} \cdot R_{OC} \cdot R_{HR} \cdot e^{-2\alpha l} = 1 , \qquad (9.6)$$

where the losses due to scattering and absorption are lumped into the distributed loss coefficient α.

In general we expect the gain to increase as we pump more energy into the laser medium. At low pump powers, the gain will be small, and there will be insufficient gain to reach the oscillation condition. The laser will not start to oscillate until there is enough gain to overcome all the losses. This implies that the laser will have a *threshold* in terms of the pump power.

Stimulated emission

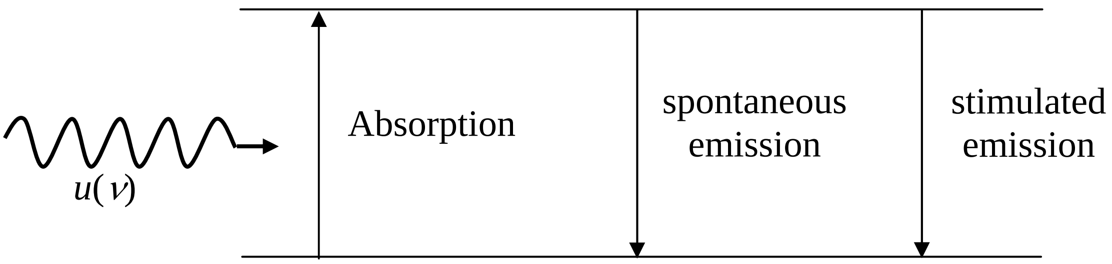
In Atomic Physics Notes 3, we considered the spontaneous tendency for atoms in excited states to emit radiation. We now consider the optical transitions that occur when the atom is subjected to electromagnetic radiation with its frequency resonant with the energy difference of the two levels. We follow the treatment of Einstein (1917).

In addition to spontaneous transitions from the upper to the lower level, there will also be:

- *Absorption* of photons causing transitions from level 1 up to level 2
- *Stimulated emission* in which atoms in level 2 drop to level 1 induced by the incident radiation.

The process of stimulated emission is a coherent quantum mechanical effect. The photons emitted by stimulated emission are in phase with the photons that induce the transition. This is the fundamental basis of laser operation, as the name suggests (Light Amplified by *Stimulated Emission* of Radiation.)

Consider an atom irradiated by white light, with N_2 atoms in level 2 and N_1 atoms in level 1. The part of spectrum at frequency ν, where $h\nu = (E_2 - E_1)$, can induce absorption and stimulated emission transitions. We write the spectral energy density of the light at frequency ν as $u(\nu)$. The transitions occurring are shown in the diagram below.



In order to treat this situation, Einstein introduced his *A* and *B* coefficients. We have already seen in Atomic Physics Notes 3 that the *A* coefficient determines the rates of spontaneous transitions. The introduction of the *B* coefficient extends the treatment to include absorption and stimulated emission.

Referring to the diagram above, the transition rates for three processes are:

Spontaneous emission	$(2\rightarrow 1)$	$\mathrm{d}N_2/\mathrm{d}t = -\mathrm{d}N_1/\mathrm{d}t = -A_{21} N_2$
Stimulated emission	$(2\rightarrow 1)$	$\mathrm{d}N_2/\mathrm{d}t = -\mathrm{d}N_1/\mathrm{d}t = -B_{21} N_2\, u(\nu)$
Absorption	$(1\rightarrow 2)$	$\mathrm{d}N_1/\mathrm{d}t = -\mathrm{d}N_2/\mathrm{d}t = -B_{12} N_1\, u(\nu)$.

These are effectively the definitions of the Einstein *A* and *B* coefficients.

<u>Relationship between the Einstein Coefficients</u>

At this stage we might be inclined to think that the three coefficients are independent parameters for a particular transition. This is not in fact the case. If you know one, you can work out the other two. To see this, we follow Einstein's analysis.

We imagine that the atom is inside a box at temperature *T* with black walls. The atom will then be bathed in black body radiation. If we leave the atom for long enough, it will come to equilibrium with the black body radiation. In these steady-state conditions, the rate of upward transitions must exactly balance the rate of downward transitions. Thus we must have :

$$B_{12} N_1 u(\nu) = A_{21} N_2 + B_{21} N_2 u(\nu) \quad . \qquad (9.7)$$

In thermal equilibrium at temperature *T*, the ratio of N_2 to N_1 is given by Boltzmann's law:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left(-\frac{h\nu}{kT}\right), \tag{9.8}$$

where g_2 and g_1 are the degeneracies of levels 2 and 1 respectively, and $h\nu = (E_2 - E_1)$.

The spectral energy density of a black body source is given by the Planck formula (See Appendix on black-body radiation) :

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \quad \frac{1}{\exp\left(h\nu/kT\right) - 1} . \tag{9.9}$$

The only way that equations (9.7) – (9.9) can be consistent with each other at all temperatures is if

$$\begin{aligned} g_1 B_{12} &= g_2 B_{21} \\ \frac{A_{21}}{B_{21}} &= \frac{8\pi h\nu^3}{c^3} \cdot \end{aligned} \tag{9.10}$$

A moment's thought will convince us that it is not possible to get consistency between the equations without the stimulated emission term. This is what led Einstein to introduce the concept.

The relationships between the Einstein coefficients given in Eq. (9.10) have been derived for the case of the atom sitting there bathed in black body radiation. However, once we have derived the inter-relationships, they will apply in all other cases as well. This is very useful, because we only need to know one of the coefficients to work out the other two. For example, we can measure the radiative lifetime to determine *A*, and then work out the *B* coefficients using Eq. (9.10).

Equation (9.10) shows that the probabilities for absorption and emission are the same apart from the degeneracy factors, and that the ratio of the probability for spontaneous emission to stimulated emission increases in proportion to ν^3. In a laser we want to encourage stimulated emission and suppress spontaneous emission. Hence is gets progressively more difficult to make lasers work as the frequency increases, all other things being equal.

Reading

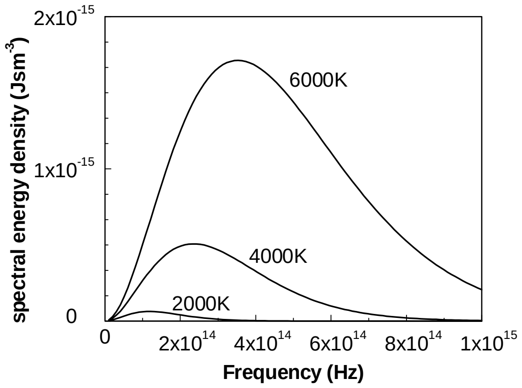
Demtröder, *Atoms, Molecules and Photons* (2006): § 7.1, 8.1
Wilson and Hawkes, *Optoelectronics* 3rd edition (1998): § 5.1 – 2 , § 6.5
Smith and King , *Optics and Photonics* (2000): chapters 15, 17
Hecht, *Optics* 4th edition (2002): §7.4.3, 12.1, 13.1
Silfvast, *Laser Fundamentals* (1996): Chapter 1, § 2.4, chapter 6
Svelto, *Principles of Lasers,* 4th edition (1998): Chapter 1, § 2.1 – 4

Appendix: Black Body Radiation

The emission of radiation from hot bodies was extensively studied in the 19th century using the laws of thermodynamics, and then consequently by quantum theory. It was the unsolved problem of explaining the spectrum from hot bodies that led Planck to propose that energy was quantized in 1901.

All hot objects emit radiation and also absorb radiation emitted by the surrounding objects. In thermodynamic equilibrium, the amount of energy absorbed must exactly balance the amount of energy emitted, unless the object has an internal energy source that accounts for the difference e.g. the sun or a light bulb. Some objects absorb and emit better than others. "Black bodies" absorb everything that hits them by definition. Shiny objects are less good at absorbing (and emitting: see 1 below).

Experimental measurements show that the spectrum emitted by a hot object is determined only by the temperature *T*. This gives rise to the concept of black body radiation with the spectrum shown below:



Here is a summary of the classical laws of the black body radiation:

1. The ratio of the spectral emissive power to the spectral absorptivity for all bodies is a universal function of wavelength and temperature only. (Kirchoff's Law).

2. The total power radiated is proportional to the fourth power of the temperature. (Stefan's Law):

$$W = \sigma T^4 \ ,$$

where W is the power emitted per unit area and σ is Stefan's constant (5.67×10^{-8} $\mathrm{Jm^{-2}s^{-1}K^{-4}}$).

3. The wavelength of the peak in the spectrum is inversely proportional to the inverse of the temperature (Wien's displacement law):

$$\lambda_{\max} T = 2.9 \times 10^{-3} \text{ m K} \ .$$

Thermodynamics can explain these results, but cannot explain the detailed shape of the energy spectrum. This requires quantum theory. In 1901 Planck derived the following formula for the spectral energy density $u(\nu,T)$ of the black body radiation:

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \quad \frac{1}{\exp\left(h\nu/kT\right) - 1} \ .$$

$u(\nu)\mathrm{d}\nu$ is the energy per unit volume of black body radiation with frequency between ν and $\nu+\mathrm{d}\nu$. This result exactly matches the experimental data. Stefan's and Wien's laws can be derived from it. It was a great triumph for Planck, and heralded the dawn of quantum theory.

<u>Reading</u>

F. Mandl, *Statistical Physics*, chapter 10

E. Hecht, *Optics* 3rd edition (1998), §13.1.1

R. Eisberg and R. Resnick, *Quantum Physics of atoms, molecules, solids, nuclei, and particles*, chap. 1

10. Laser gain mechanisms

Population inversion

In Laser Notes 1 we introduced the notion of *stimulated emission*, which is the basis of laser operation. We now wish to study how we can use stimulated emission to make a light amplifier. In a gas of atoms in thermal equilibrium, the population of the lower level will always be greater than the population of the upper level. (See Eq. 9.8). Therefore, if a light beam is incident on the medium, there will always be more upward transitions due to absorption than downward transitions due to stimulated emission. (See Fig. 10.1.) Hence there will be net absorption, and the intensity of the beam will diminish on progressing through the medium.

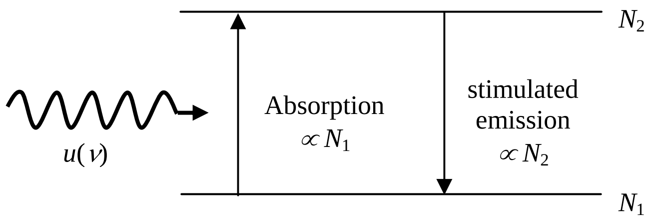


Figure 10.1
Absorption and stimulated emission transitions.

To amplify the beam, we require that the rate of stimulated emission transitions exceeds the rate of absorption. If the light beam is sufficiently intense that we can ignore spontaneous emission[1] and the levels are non-degenerate, this implies that N_2 must exceed N_1. This is a highly non-equilibrium situation, and is called *population inversion*. Inspection of Eq. 9.8 implies that population inversion corresponds to negative temperatures ! This is not as ridiculous as it sounds, because the atoms are not in thermal equilibrium.

Once we have population inversion, we have a mechanism for generating gain in the laser medium. The art of making a laser operate is to work out how to get population inversion for the relevant transition.

Gain

Having realised that population inversion gives us a gain mechanism, we now want to work out a relationship between the gain coefficient and the inversion density. Before we can proceed, we must first refine our analysis of the absorption and stimulated emission rates. Einstein's analysis considered the interaction of an ideal atom with a featureless white light spectrum. In practice, we are more interested in the interaction of real atoms with sharp emission lines with an even narrower band of light that will eventually become the laser mode.

The energy density $u(\nu)$ that appears in eqns 9.7 – 9 is the *spectral* energy density (units: $\mathrm{Jm}^{-3}/\mathrm{Hz} \equiv \mathrm{Jsm}^{-3}$). We now consider the interaction between an atom with a normalised line shape function $g(\nu)$ (units: Hz^{-1}) as defined in Atomic Physics Section 3.7 and a beam of light whose emission spectrum is much narrower than the spectral line width of the atomic transition. In this case, the rates of absorption and stimulated emission can be written respectively in following form (see Wilson and Hawkes, Optoelectronics (1998), Appendix 4):

$$\begin{aligned} W_{12} &= B_{12} N_1 \mathrm{u}_\nu\, g(\nu) \\ W_{21} &= B_{21} N_2 \mathrm{u}_\nu\, g(\nu) \end{aligned} . \qquad (10.1)$$

The light source is considered to have a delta function spectrum at frequency ν with *total* energy density u_ν per unit volume (units Jm^{-3}). u_ν is related to the intensity I of the optical beam by (see Fig. 10.2 (a)):

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[1] We usually ignore spontaneous emission throughout this set of notes because we are considering the case in which the light intensity is very high, so that the stimulated emission rate far exceeds the spontaneous emission rate.

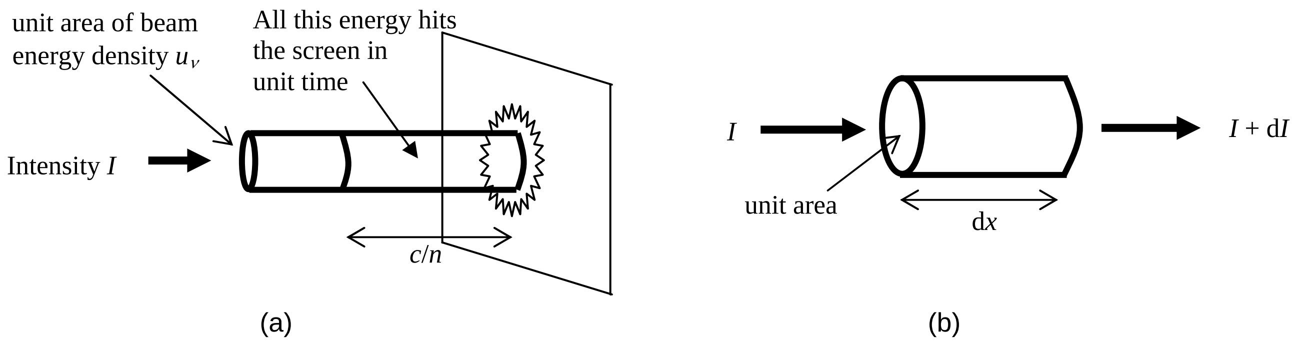


Figure 10.2. (a) Relationship between the intensity I and energy density u_ν of a light beam. (b) Incremental intensity increase in a gain medium.

$$I = u_\nu \frac{c}{n} \ , \tag{10.2}$$

where n is the refractive index of the medium. This means that the net stimulated rate downwards from level 2 to level 1 is given by

$$W_{21}^{net} = (N_2 - N_1)\, B_{21}\, g(\nu) \frac{n}{c} I \tag{10.3}$$

where we have assumed that the levels are non-degenerate so that $B_{12} = B_{21}$ (cf Eq. 9.10).

For each net transition a photon of energy $h\nu$ is added to the beam. The energy added to a unit volume of beam per unit time is thus $W_{21}^{net}\, h\nu$. Consider a small increment of the light beam inside the gain medium with length dx, as shown in Fig. 10.2(b). The energy added to this increment of beam per unit time is $W_{21}^{net}\, h\nu\, dx \times$ (area of beam). Remembering that the intensity equals the energy per unit time per unit area, we can write

$$\begin{aligned} dI &= W_{21}^{net}\, h\nu\, dx \\ &= (N_2 - N_1)\; B_{21}\, g(\nu)\, \frac{n}{c}\, h\nu\, I\, dx \end{aligned} \ . \tag{10.4}$$

On comparing this to equation (9.4), we see that the gain coefficient γ is given by:

$$\gamma(\nu) = (N_2 - N_1)\; B_{21}\, g(\nu)\, \frac{n}{c}\, h\nu \, . \tag{10.5}$$

This result shows that the gain is directly proportional to the population inversion, and also follows the spectrum of the emission line. By using equation (9.10) to express B_{21} in terms of A_{21}, we can re-write the gain coefficient in terms of the natural lifetime τ ($= A_{21}^{-1}$) as (See Atomic Physics eqn 3.25):

$$\gamma(\nu) = (N_2 - N_1)\, \frac{\lambda^2}{8\pi n^2 \tau}\, g(\nu) \ , \tag{10.6}$$

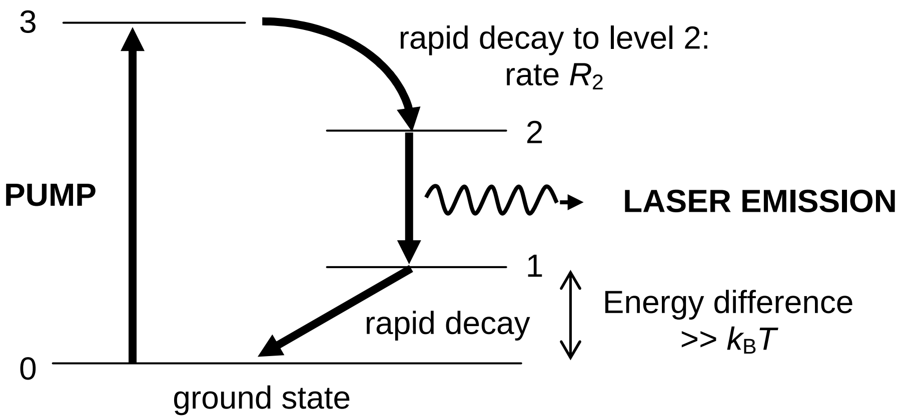


Figure 10.3
Level scheme for a four-level laser.

where λ is the wavelength of the emission line. This is the required result. Eq. 10.6 tells us how to relate the gain in the medium to the population inversion using experimentally measurable parameters: λ, τ, n and $g(\nu)$.

Laser threshold

We have seen above in Eq. (10.6) that the gain in a laser medium is directly proportional to the population inversion. Laser operation will occur when there is enough gain to overcome the losses in the cavity. This implies that a minimum amount of population inversion must be obtained before the laser will oscillate. Population inversion is achieved by "pumping" atoms into the upper laser level. This pumping can be done by a variety of techniques, which will be described in more detail when we consider individual laser systems in Laser Notes 12. At this stage we just consider the general principles.

Lasers are classified as being either *three-level* of *four-level* systems. We will consider four-level lasers first, as these are the most common. Examples are Helium Neon or Nd:YAG. The four levels are: the ground state (0), the two lasing levels (1 & 2), and a fourth level (3) which is used as part of the pumping mechanism. The level scheme for an ideal four-level laser is shown in Fig. 10.3. The feature that makes it a four-level as opposed to a three-level laser is that the lower laser level is at an energy more than k_BT above the ground state. This means that the thermal population of level 1 is negligible, and so level 1 is empty before we turn on the pumping mechanism.

We assume that the atoms are inside a cavity and are being pumped into the upper laser level (level 2) at a constant rate of R_2 . This is usually done by exiting atoms to level 3 with a bright flash lamp or by an electrical discharge, and then by a rapid decay to level 2.

We can write down the following rate equations for the populations of levels 1 and 2:

$$\begin{aligned} \frac{\mathrm{d}N_2}{\mathrm{d}t} &= -\frac{N_2}{\tau_2} - W_{21}^{net} + R_2 \\ \frac{\mathrm{d}N_1}{\mathrm{d}t} &= +\frac{N_2}{\tau_2} + W_{21}^{net} - \frac{N_1}{\tau_1} \end{aligned} \quad . \tag{10.7}$$

The various terms allow for:

- spontaneous emission from level 2 to level 1 ($\pm N_2/\tau_2$),
- stimulated transitions from level 2 to level 1 ($\pm W_{21}^{net}$),
- pumping into level 2 (R_2),
- decay from level 1 to the ground state by radiative transitions and/or collisions (N_1 / τ_1).

Note that W_{21}^{net} is the *net* stimulated transition rate from level 2 to level 1. This is equal to the rate of stimulated emission transitions downwards minus the rate of stimulated absorption transitions upwards.

There are two important assumptions implicitly contained in Eq. (10.7):

1. There is no pumping into level 1.
2. The only decay route from level 2 is by radiative transitions to level 1 (ie there are no non-radiative transitions between level 2 and level 1, and transitions to other levels are not possible).

It may not always be possible to arrange these conditions, but it helps if we can. That's why we described the above scenario as an "ideal" four-level laser.

We can re-write Eq. (10.3) in the following form:

$$W_{21}^{net} = B_{21}\, g(\nu) \frac{n}{c} I\ (N_2 - N_1)\ =\ W(N_2 - N_1)\ , \tag{10.8}$$

where $W = B_{21}\ g(\nu)\ I\ n\ /\ c$, and I is the intensity inside the laser cavity. In steady-state conditions, the time derivatives in Eq. (10.7) must be zero. We can thus solve Eq. (10.7) for N_1 and N_2 using Eq. (10.8) to obtain:

$$N_1 = R_2\, \tau_1$$

$$N_2 = \frac{WN_1 + R_2}{W + 1/\tau_2}\quad . \tag{10.9}$$

Therefore the population inversion is given by

$$\Delta N = N_2 - N_1 = \frac{R_2}{W + 1/\tau_2}\left(1 - \frac{\tau_1}{\tau_2}\right)\ . \tag{10.10}$$

This shows that it is not possible to achieve population inversion unless $\tau_2 > \tau_1$. This makes sense if you think about it. Unless the lower laser level empties quickly, atoms will pile up in the lower laser level and this will destroy the population inversion.

Equation (10.10) can be re-written as :

$$\Delta N = \frac{R}{W + 1/\tau_2}\ , \tag{10.11}$$

where $R = R_2\,(1 - \tau_1/\tau_2)$. This is the net pumping rate after allowing for the unavoidable accumulation of atoms in the lower level because τ_1 is non-zero. If the laser is below the threshold for lasing, there will be very few photons in the cavity. Therefore, W will be very small because I is very small: see Eq. (10.8) above. The population inversion is simply $R\tau_2$, and thus increases linearly with the pumping rate. Equation (10.6) implies that the gain coefficient similarly increases linearly with the pumping rate below threshold.

Eventually we will have enough gain to balance the round trip losses. This determines the threshold gain coefficient $\gamma^{\text{threshold}}$ for laser oscillation, as set out in Eq. (9.5) or (9.6). From equation (10.6) we have:

$$\Delta N^{\text{threshold}} = \frac{8\pi n^2 \tau_2}{\lambda^2 g(\nu)} \gamma^{\text{threshold}}\ . \tag{10.12}$$

By combining equations (1011) and (102) with $W = 0$ we can work out the pumping rate required to obtain lasing. This is the *threshold* pumping rate. It is given by $R^{\text{threshold}} = \Delta N^{\text{threshold}} / \tau_2$. All lasers have a threshold. If you don't pump them hard enough, they won't go.

What happens if we increase the pumping rate beyond the threshold value? In steady state conditions, the gain cannot increase any more. This is known as *gain saturation*. It implies that the population inversion is clamped at the value given by equation (10.12) even when R exceeds $R^{\text{threshold}}$. This is shown in Fig. 10.4(a) below.

What about the power output ? We set W to zero in equation (10.11) below threshold because there was very little light in the cavity. This is no longer true once the laser starts oscillating. If we are above threshold, ΔN is clamped at the value set by Eq. (10.12), and so equation (10.11) tells us that:

$$\begin{aligned} W &= \frac{R}{\Delta N^{\text{threshold}}} - \frac{1}{\tau_2} \\ &= \frac{1}{\tau_2}\left(\frac{R}{R^{\text{threshold}}} - 1\right) \end{aligned} \qquad (10.13)$$

Now W is proportional to the intensity I inside the cavity (see Eq. 10.3), which in turn is proportional to the output power P^{out} emitted by the laser. Thus P^{out} is proportional to W, and we may write :

$$P^{\text{out}} \propto \frac{R}{R^{\text{threshold}}} - 1 \quad . \qquad (10.14)$$

This shows that the output power increases linearly with the pumping rate once the threshold has been achieved, as shown in Fig. 10.4(b) below.

The choice of the reflectivity of the output coupler affects the threshold because it determines the oscillation conditions: see Eq. (9.5) or (9.6). If the transmission ($1-R_{\text{OC}}$) is small, we will have a low threshold, but the output coupling efficiency will be low. By increasing the transmission, the threshold increases, but we can get the power out more efficiently once we are above threshold. This point is illustrated in Fig. 10.4(b). The final choice for R_{OC} depends on how much pump energy is available, which will govern the optimal choice to get the maximum output power.

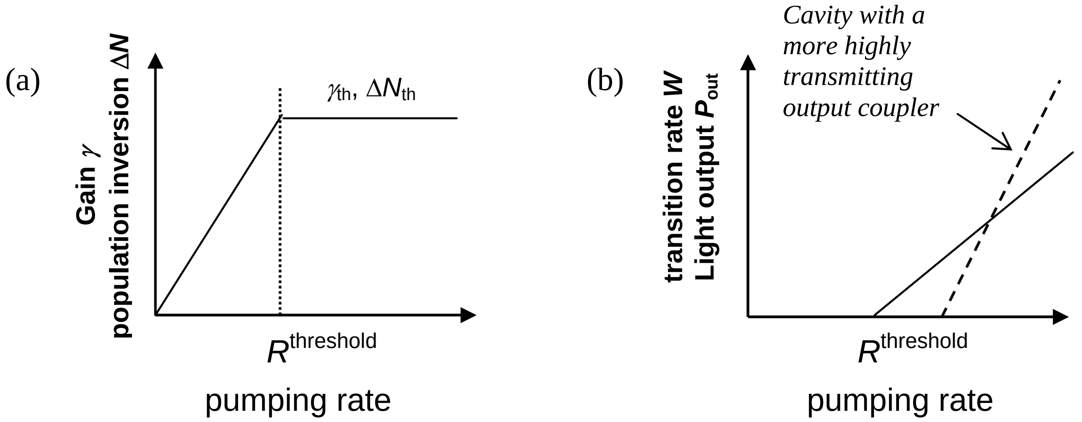


Figure 10.4 (a) Variation of the gain and population inversion in a laser with the pumping rate. (b) Comparison of the threshold and light output for two different values of the output coupler.

Pulsed Lasers

So far we have only considered continuous lasers, but many lasers in fact operate in a pulsed mode. Powerful pulsed flash lamps can give rise to very large pumping rates, with correspondingly large output pulse energies, especially when using a trick called Q-switching. In this technique, the losses in the cavity are kept artificially high by some external method (eg by inserting an intracavity shutter). This prevents lasing and allows the build up of very large population inversion densities, with correspondingly large gain coefficients. If the losses are suddenly reduced (eg by opening the shutter), a very powerful pulse will build up because of the very high gain in the cavity. Q-switching is widely used in solid state lasers because they tend to have long upper state lifetimes, which allows the storage of a large amount of energy in the crystal, but it is seldom used in gas lasers because the lifetimes are shorter which makes it difficult to store much energy in the gain medium.

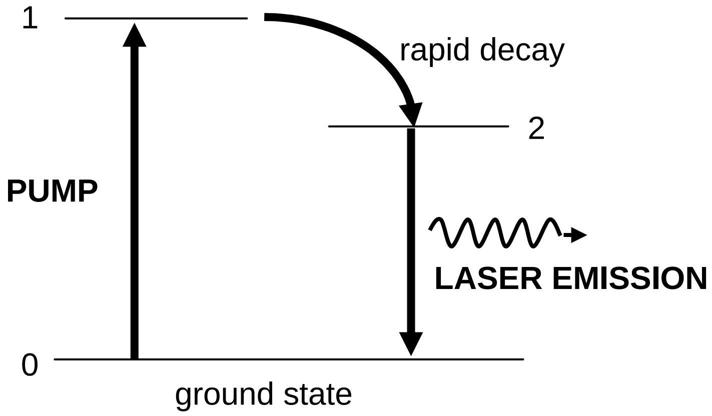


Figure 10.5

Level scheme for three-level laser, for example: ruby.

Three-Level lasers

Some lasers are classified as being *three-level* systems. The standard example is ruby, which was the first laser ever produced. The key difference between a three-level laser and a four-level laser is that the lower laser level is the ground state, as shown in Fig. 10.5. It is much more difficult to obtain population inversion in three-level lasers because the lower laser level initially has a very large population. Let this population be N_0. By turning on the pump, we excite dN atoms to level 1, which then decay to level 2. Thus the population of Level 2 will be dN, and the population of the ground state will be (N_0-dN). Hence for population inversion we require dN > (N_0-dN), that is dN > N_0/2. Therefore, in order to obtain population inversion we have to pump more than half the atoms out of the ground state into the upper laser level. This obviously requires a very large amount of energy. This contrasts with the four-level lasers in which the lower laser level is empty before the pumping process starts, and much less energy is required to reach threshold.

Despite the fact that the threshold for population inversion is very high in a three-level system, they can be quite efficient once this threshold is overcome. Ruby lasers pumped by bright flash lamps actually give very high output pulse energies. However, they only work in pulsed mode. Continuous lasers tend to be made using four-level systems.

Reading

Demtröder, *Atoms, Molecules and Photons* (2006): § 8.1
Wilson and Hawkes, *Optoelectronics* 3rd edition (1998): § 5.1 – 5.8 and appendix 4
Smith and King, *Optics and Photonics*, chapter 15
Hecht, *Optics* 4th edition (2002): §13.1
Yariv, *Optical Electronics in Modern Communications* 5th edition (1997): § 5.1 – 3, 6.3 – 6.5 *
Silfvast, *Laser Fundamentals* (1996): chapters 7 – 8 *
Svelto, *Principles of Lasers*, 4th edition (1998): § 2.1 – 4, § 7.1 – 7.3 *

* more advanced treatments, some of which is beyond the level of this course.

11. Laser cavities and modes

The cavity is an essential part of a laser. It provides the positive feedback that turns an amplifier into an oscillator. The design of the cavity is therefore very important for the optimal operation of the laser. The cavity determines the properties of the beam of light that is emitted by the laser. This beam is characterised by its *transverse* and *longitudinal* mode structure.

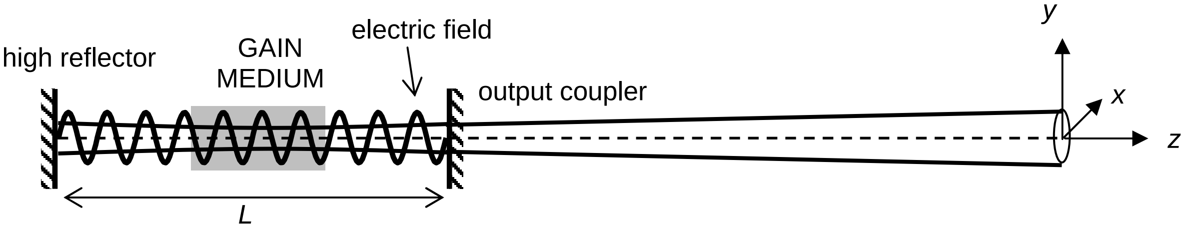


Figure 11.1 Laser cavity and output beam

Transverse modes

These describe the variation of the electrical field within a cross-sectional slice of the beam. The modes are labelled TEM_{mn} where *m* and *n* are integers. "TEM" stands for “transverse electro magnetic”. If the field is propagating in the *z* direction, the (*x*,*y*) dependence of the field is given by:

$$E_{mn}(x,y) = E_0\, H_m\left(\frac{\sqrt{2}x}{w}\right) H_n\left(\frac{\sqrt{2}y}{w}\right) \exp\left(-\frac{x^2+y^2}{w^2}\right), \tag{11.1}$$

where H_m and H_n are mathematical functions called the Hermite polynomials. The first few polynomials are:

$$H_0(u) = 1 \;;\; H_1(u) = 2u \;;\; H_2(u) = 2(2u^2-1)\,. \tag{11.2}$$

The most important mode is the TEM_{00} mode. This has a Gaussian radial distribution (see Fig. 3.2):

$$E_{00}(x,y) = E_0\ \exp\left(-\frac{x^2+y^2}{w^2}\right) = E_0\ \exp\left(-\frac{r^2}{w^2}\right), \tag{11.3}$$

where *r* is the distance from the centre of the beam. The parameter *w* that appears in these equations determines the size of the beam. The TEM_{00} mode is the closest thing to a ray of light found in nature. It has the smallest divergence of all the modes and can be focussed to the smallest size. We therefore usually try hard to prevent other modes from oscillating. This is achieved by inserting apertures in the cavity which are lossy for the higher modes but not for the smaller TEM_{00} mode.

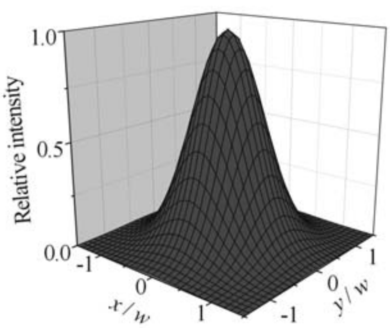


Figure 11.2
Intensity distribution of a TEM_{00} mode, which has a Gaussian profile.

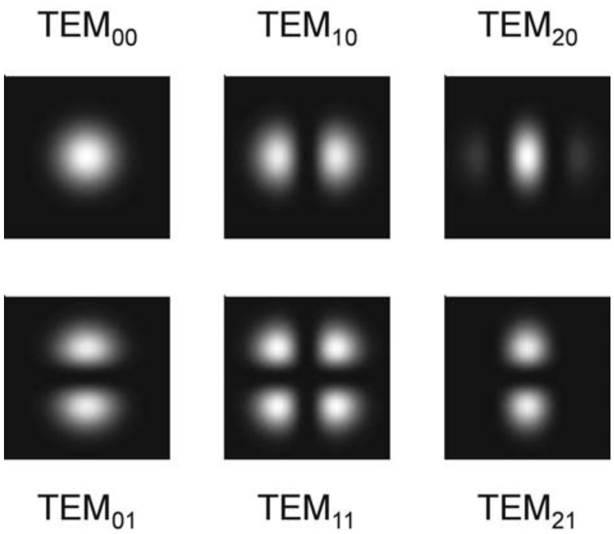


Figure 11.3
Beam profiles produced by various laser modes. Note that the side lobes in the *x* direction for the TEM_{21} mode are too faint to be seen on this grey scale.

Figure 11.3 compares the beam cross-section for a number of higher order laser modes with that of the TEM_{00} mode. Note that the TEM_{mn} mode has *m* nodes (zeros) in the *x* direction and *n* nodes in the *y* direction. These higher order modes make pretty pictures, but are not useful for very much. A well-designed laser will contain apertures that allow only the TEM_{00} mode to oscillate.

Longitudinal modes

The longitudinal modes determine the emission spectrum of the laser. The light bouncing repeatedly off the end mirrors sets up standing waves inside the cavity (see Figure 11.1). There are nodes (field zeros) at the mirrors because they have high reflectivities. Thus there must be an integer number of half wavelengths inside the cavity. If the length of the cavity is *L*, this condition can be written:

$$L = \text{integer} \times \frac{\lambda}{2} = \text{integer} \times \frac{c}{2n\nu}\ , \tag{11.4}$$

where *n* is the average refractive index of the cavity. In gas lasers, *n* will be very close to unity. It will also be the case that $n \approx 1$ in a solid state laser with a short laser rod inside a long air-filled cavity. Equation (11.4) implies that only certain frequencies that satisfy

$$\nu = \text{integer} \times \frac{c}{2nL} \tag{11.5}$$

will oscillate. Most cavities are much bigger than the optical wavelength and thus the value of the integer in equations (11.4) and (11.5) is very large.[2] The most important parameter is the longitudinal mode spacing:

$$\Delta\nu = \frac{c}{2nL}\ . \tag{11.6}$$

The Table below lists the longitudinal mode spacing for several lasers.

	Diode laser	HeNe laser	Argon ion laser
L	1 mm	30 cm	2 m
n	3.5	1	1
Mode spacing (Hz)	1.5×10^{11}	5×10^{8}	7.5×10^{7}

[2] This is not true for "microlasers", where we deliberately make the cavity to be of similar dimensions to the optical wavelength. In this case the integer would have a value of 1. Some semiconductor lasers used in optical fibre systems are small enough to operate as microlasers.

Multi-mode and single-mode lasers

For a given mode to oscillate, its frequency must lie within the emission spectrum of the laser transition. Unless we do something about it, there will be a tendency for all the longitudinal modes which experience gain to oscillate. Therefore the laser will have *multi-mode* operation, as illustrated in Fig. 11.4 below. As a rough guide, the number of modes that will be oscillating is equal to the gain bandwidth divided by the mode spacing. Thus for a 30 cm HeNe laser with a gain bandwidth of 1.5 GHz, there will be three modes oscillating. In a Doppler-broadened emission line such as that from the Neon atoms in a HeNe laser, the phases of these modes will be random relative to each other because they are emitted by different atoms.

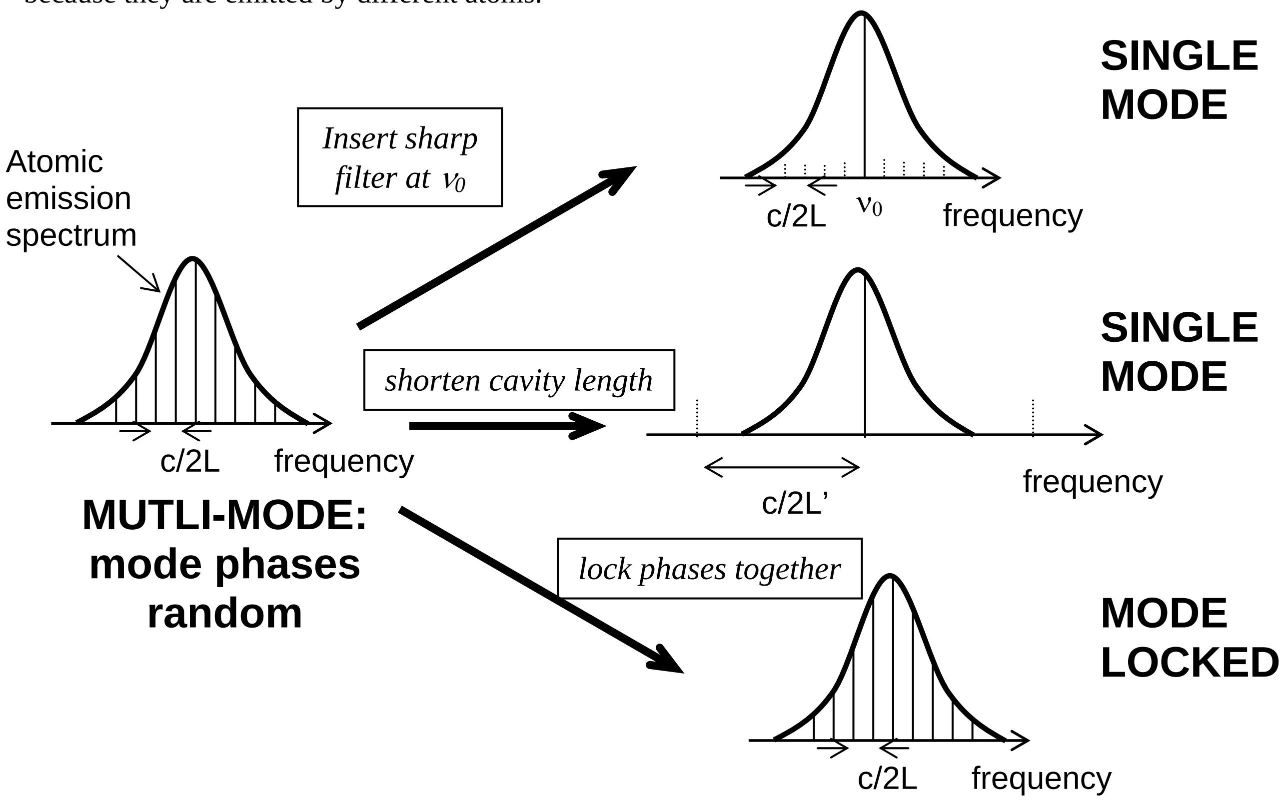


Figure 11.4: Multi-mode, single mode and mode-locked operation

Several applications require narrow line width emission from the laser. Examples include high resolution spectroscopy, holography and interferometry (see discussion on coherence in Laser Physics 9). It is therefore interesting to see if we can make the laser go on just a single longitudinal mode. The spectral linewidth would then be determined by the Q of the cavity rather than the gain band width. This is called *single-mode* operation.

One way to achieve to single-mode operation is to shorten the cavity so that the mode spacing exceeds the gain bandwidth. See Fig. 11.4 above. In this case only one mode will fall within the emission line of the transition and the laser will automatically oscillate on only one mode. However, this may not be practical. For example, in the case of the HeNe laser considered above, we would need to make the cavity shorter than 10 cm. Such a laser would have very small round-trip gain, and we would probably not be able to make it oscillate. A better way to obtain single mode operation is to insert a narrow frequency filter in the cavity such as a Fabry-Perot etalon. By tuning the spacing of the etalon, the frequency of the single mode can be changed continuously, which is very useful for high resolution spectroscopy. The spectral line width of a single-mode laser is typically a few MHz. This is about a thousand times narrower than the atomic emission line that produces the light.

Mode-locking

The opposite extreme to single-mode operation is *mode-locked* operation. In this situation we try to get as many longitudinal modes oscillating as possible, but with all their phases locked together. (See Fig. 11.4). This contrasts with a multi-mode laser in which many modes are oscillating but with random phases with respect to each other.

In the Appendix we prove that the mode-locked operation of a laser corresponds to a single pulse oscillating around the cavity and getting emitted every time it hits the output coupler, as shown in Fig. 11.5 below. The time taken for a pulse to circulate around a cavity with n = 1of length L is $2L/c$. Therefore we get pulses out of the laser at a repetition rate of $(2L/c)^{-1}$.

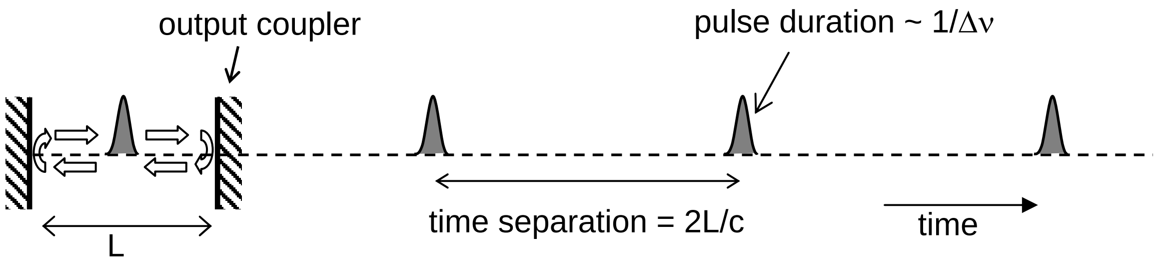


Figure 11.5   Mode-locked laser pulses from a cavity with n = 1.

The minimum pulse duration is set by the Fourier transform of the spectrum:

$$\Delta t_{\min} \Delta \nu \gtrsim 1/2\pi \quad . \tag{11.7}$$

This "uncertainty principle" means that to get very short pulses we need a wide gain bandwidth. Gas lasers are not very good in this context because they are based on fairly narrow atomic transitions. The best results have been achieved in tuneable lasers such as dye lasers or titanium doped sapphire lasers. The gain bandwidth of Ti:sapphire is nearly 10^{14} Hz, and pulses as short as 10 fs have been produced from this laser. This corresponds to millions of longitudinal modes oscillating.

Mode-locking is achieved by two main techniques. With active mode-locking, a time-dependent shutter is inserted in the cavity. The shutter is opened briefly every $2L/c$ seconds. Continuous operation of the laser is impossible, but the mode-locked pulses will be unaffected by the shutter. In passive mode-locking, a saturable absorber is inserted in the cavity. Such absorbers have strong absorption at low powers and small absorption at high powers. The peak power in the pulsed mode is much higher than in continuous operation, and thus the cavity naturally selects the pulsed mode.

Mode-locked lasers are widely used in scientific research to study fast processes in physics, chemistry, and biology. For example, the typical time for a current-carrying electron in a copper wire to interact with a phonon at room temperature is about 100 fs. Similarly, the early stages of many chemical reactions or biological processes such as photosynthesis take place in less than 10^{-12} s. Mode-locked lasers are also useful to telecommunication companies, who are interested in packing as many bits of information (represented by pulses of light) as possible down their optical fibres. The shorter the pulses, the higher the data rate. There are also medical applications: it is much cleaner to use a very short, low-energy, high-peak-power pulse for laser surgery, than a longer pulse with the same peak power but much higher energy.

Reading

Demtröder, *Atoms, Molecules and Photons* (2006): § 8.2–8.3, 8.6
Wilson and Hawkes, *Optoelectronics* 3rd edition (1998): § 5.9, 6.1–6.3
Smith and King, Optics and Photonics, §15.7, 17.4
Hecht, *Optics* 4th edition (2002): §13.1
Yariv, *Optical Electronics in Modern Communications* 5th edition (1997): §6.6–7 *
Silfvast, *Laser Fundamentals* (1996): chapter 10, 12.3 *
Svelto, *Principles of Lasers*, 4th edition (1998): § 7.7–8, 8.6 *

* more advanced treatments, some of which is beyond the level of this course.

Appendix: Mode locked emission

The electric field of the light emitted by a multi-mode laser is given by:

$$E(t) = \sum_m E_m \exp\left(\mathrm{i}\omega_m t + \varphi_m\right), \tag{11.8}$$

where the sum is over all the longitudinal modes that are oscillating. ω_m is the angular frequency of the m^{th} mode ($= m\pi c/L$, for $n = 1$). In multi-mode operation all the phases of the modes are different, and not much can be done with the summation. However, in a mode-locked laser, all the phases are the same (call it φ_0) because they have been locked together. This allows us to evaluate the summation.

We assume that all the modes have approximately equal amplitudes E_0. The output field of the mode-locked laser is then given by:

$$E(t) = E_0\, \mathrm{e}^{\mathrm{i}\varphi_0} \sum_m \mathrm{e}^{\mathrm{i}\omega_m t} \; . \tag{11.9}$$

Let's suppose there are N modes oscillating, and the frequency of the middle mode is ω_0. This gives the field as :

$$\begin{aligned} E(t) &= E_0 \;\; \mathrm{e}^{\mathrm{i}\varphi_0} \sum_{m'=-(N-1)/2}^{m'=+(N-1)/2} \exp\left(\mathrm{i}(\omega_0 + \frac{m'\pi c}{L})t\right) \\ &= E_0 \;\; \mathrm{e}^{\mathrm{i}\varphi_0} \mathrm{e}^{\mathrm{i}\omega_0 t} \sum_{m'=-(N-1)/2}^{m'=+(N-1)/2} \exp\left(\mathrm{i}\,\frac{m'\pi c}{L}\,t\right) \end{aligned} \; . \tag{11.10}$$

This type of summation is frequently found in the theory of diffraction gratings. It can be evaluated by standard techniques [3]. We are actually interested in the time dependence of the output power, which is given by:

$$P(t) \propto E(t)\; E(t)^* . \tag{11.11}$$

The final answer is:

$$P(t) \propto \frac{\sin^2(N\pi ct/2L)}{\sin^2(\pi ct/2L)} \; . \tag{3.12}$$

This function has big peaks whenever t = integer×$2L/c$ and is small at all other times. Thus the output consists of pulses separated in time by $2L/c$. The duration of the pulse is approximately given by the time for the numerator to go to zero after one of the major peaks. This time is $2L/Nc$. The frequency band width is equal to the (number of modes oscillating) × (spacing between modes), ie $\Delta\nu = N \times c/2L$. Thus $\Delta t \Delta\nu = 1$, as expected from the uncertainty principle.

---

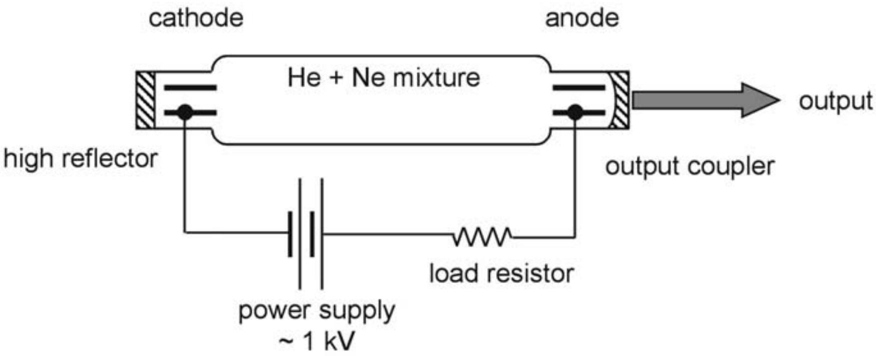
[3] Remember that $e^{ab} = (e^a)^b$, and that $\sum_{j=0}^{n-1} r^j = \dfrac{1-r^n}{1-r}$

12. Different types of lasers

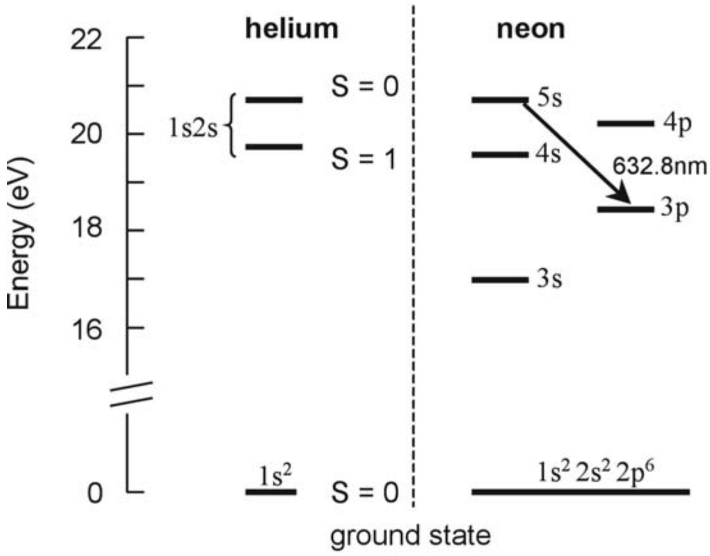
Lasers are generally classified by their phase, that is: solid, liquid or gas. The main example of a liquid laser is the dye laser. We do not consider dye lasers here, but just concentrate on gas and solid-state lasers, starting with gas lasers.

GAS LASERS

The Helium Neon (HeNe) Laser



Helium neon lasers consist of a discharge tube inserted between highly reflecting mirrors. The tube contains a mixture of helium and neon atoms in the approximate ratio of He:Ne 5:1. By applying a high voltage across the tube, an electrical discharge can be induced. The electrons collide with the atoms and put them in an excited state. The light is emitted by the neon atoms, and the purpose of the helium is to assist the population inversion process. To see how this works we need to refer to the level diagram below.



Helium has two electrons. In the ground state both electrons are in the 1s level. The first excited state is the 1s2s configuration. There are two possible energies for this state because there are two possible configurations of the electron spin: the singlet $S = 0$ and the triplet $S = 1$ terms. The helium atoms are excited by collisions with the electrons in the discharge tube and cascade down the levels. When they get to the 1s2s configuration, however, the cascade process slows right down. In the $2s1s \Rightarrow 1s^2$ transition one of the electrons jumps from the 2s level to the 1s level. This is forbidden by the $\Delta l = \pm 1$ selection rule. Furthermore transitions from the $S = 1$ level to the other $S = 0$ levels (including the ground state which is an $S = 0$ level) are forbidden by the $\Delta S = 0$ selection rule. The net result is that all transitions from the 1s2s levels are strongly forbidden. The 1s2s level is therefore called *metastable*. See Atomic Physics Notes 8 for more details.

Neon has ten electrons in the configuration $1s^2 2s^2 2p^6$. The excited states correspond to the promotion of one of the 2p electrons to higher levels. This gives the level scheme shown in the diagram. The symbols of the excited states refer to the level of this single excited electron. By good luck, the 5s and 4s levels of the neon atoms are almost degenerate with the S = 0 and S = 1 terms of the 1s2s configuration of helium. Thus the helium atoms can easily de-excite by collisions with neon atoms in the ground state according to the following scheme:

$$He^* + Ne \Rightarrow He + Ne^* .$$

The star indicates that the atom is an excited state. Any small differences in the energy between the excited states of the two atoms are taken up as kinetic energy.

This scheme leads to a large population of neon atoms in the 5s and 4s excited states. This gives population inversion with respect to the 3p and 4p levels. It would not be easy to get this population inversion without the helium because collisions between the neon atoms and the electrons in the tube would tend to excite all the levels of the neon atoms equally. This is why there is more helium than neon in the tube.

The main laser transition at 632.8 nm occurs between the 5s level and the 3p level. The lifetime of the 5s level is 170 ns, while that of the 3p level is 10 ns. This transition therefore easily satisfies the criterion given in Eq. 2.10, namely $\tau_{upper} > \tau_{lower}$. This ensures that atoms do not pile up in the lower level once they have emitted the laser photons, as this would destroy the population inversion. The atoms in the 3p level rapidly relax to the ground state by radiative transitions to the 3s level and then by collisional de-excitation to the original 2p level.

Lasing can also be obtained on other transitions: for example, 5s $\Rightarrow$ 4p at 3391 nm and 4s $\Rightarrow$ 3p at 1152 nm. These are not as strong as the main 632.8 nm line.

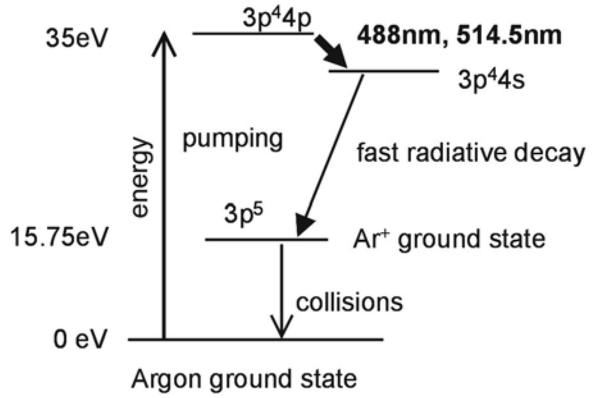
The gain in a HeNe tube tends to be rather low because of the relatively low density of atoms in the gas (compared to a solid). This is partly compensated by the fairly short lifetime of 170 ns (see Eq. 10.6). The round trip gain may only be a few percent, and so very highly reflecting mirrors are needed. With relatively small gain, the output powers are not very high – only a few mW. However, the ease of manufacture makes these extremely common lasers for low power applications: bar-code readers, laser alignment tools (theodolites, rifle sights), classroom demos etc. They are gradually being replaced nowadays by visible semiconductor laser diodes, which are commonly used in laser pointers.

Related lasers: Helium cadmium

The population inversion scheme in HeCd is similar to that in HeNe except that the active medium is Cd^+ ions. The laser transitions occur in the blue and the ultraviolet at 442 nm, 354 nm and 325 nm. The UV lines are useful for applications that require short wavelength lasers, such as high precision printing on photosensitive materials. Examples include lithography of electronic circuitry and making master copies of compact disks.

Argon ion (Ar^+) Lasers

Argon has 18 electrons with the configuration $1s^2 2s^2 2p^6 3s^2 3p^6$. Argon atoms incorporated into a discharge tube can be ionized by collisions with the electrons. The Ar^+ ion has 17 electrons. The excited states of the Ar^+ ion are generated by exciting one of the five 3p electrons to higher levels. The level scheme is given below. The important transitions occur between the 4p and 4s levels of the Ar^+ ion. Due to fine structure (spin-orbit coupling) this is actually a doublet. The two emission lines are at 488 nm (blue) and 514.5 nm (green). Several other visible transitions are also possible, making Ar^+ lasers very good for colourful laser light shows.



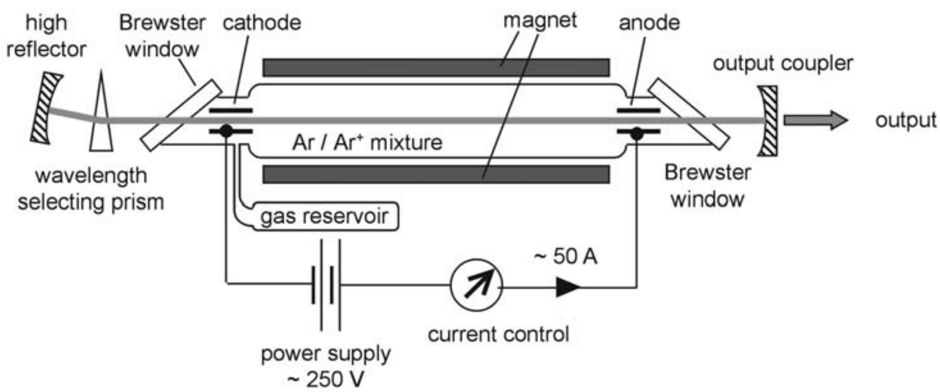
Population inversion is achieved in a two-step process. First of all, the electrons in the tube collide with argon atoms and ionize them according to the scheme:

Ar (ground state) + lots of energetic electrons
$\Rightarrow$
Ar^+ (ground state) + (lots + 1) less energetic electrons .

The Ar^+ ground state has a long lifetime and some of the Ar^+ ions are able to collide with more electrons before recombining with slow electrons. This puts them into the excited states according to:

Ar^+ (ground state) + high energy electrons $\Rightarrow$ Ar^+ (excited state) + lower energy electrons .

Since there are six 4p levels as compared to only two 4s levels, the statistics of the collisional process leaves three times as many electrons in the 4p level than in the 4s level. Hence we have population inversion. Moreover, cascade transitions from higher excited states also facilitates the population inversion mechanism. The lifetime of the 4p level is 10 ns, which compares to the 1 ns lifetime of the 4s level. Hence we satisfy $\tau_{upper} > \tau_{lower}$ and lasing is possible.



The diagram above shows a typical arrangement used in an Ar^+ laser. Argon lasers tend to be much bigger than helium-neons. The tube length might be 1–2 m, and the tube might be running at 50 A with a voltage of 250 V. Hence water-cooling is usually necessary. Output powers up to several tens of Watts are possible. The tube is enclosed in a magnet to constrain the Ar^+ ions and protect them from deflections by stray fields. The windows at the ends of the tube are cut at Brewster's angle (which satisfies $\tan\theta = n$) to reduce refection losses. There is no reflected beam for vertically polarized light at this angle. Since there are several laser transitions with similar wavelengths, it is necessary to use a prism to select the emission line that is to be used.

In addition to laser light shows, argon lasers are used for pumping tunable lasers such as dye lasers and Ti:sapphire lasers. There are also some medical applications such as laser surgery, and scientific applications include fluorescence excitation and Raman spectroscopy.

Related laser: Krypton ion $Kr^{\pm}$

Krypton ion lasers work by very similar principles to the argon laser. All you have to do is change the gas in the discharge tube and choose the mirror reflectivities to match the emission lines. The strongest line is in the red at 676.4 nm. Some laser manufacturers sell lasers with a mixture of Ar^+ and Kr^+ in the tube. We then get emission at 676 nm (red), 514 nm (green) and 488 nm (blue): this is great for laser light shows.

Carbon dioxide Lasers

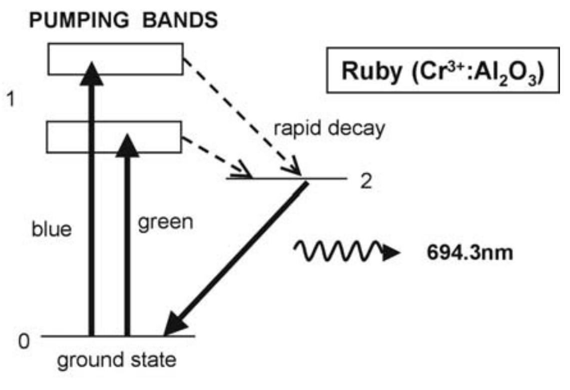
The CO_2 laser is one of the best examples of a molecular laser. The transitions take place between the vibrational levels of the molecule. The strongest emission lines are in the infrared around 10.6 μm. The lasers are very powerful with powers up to several kilowatts possible. Hence they are used in cutting applications in industry (including the military industry!) and also for medical surgery. The high power output is a consequence of the fact that the stimulated emission becomes more favourable compared to spontaneous emission at lower frequencies (See Eq. 9.10).

A mixture of nitrogen and CO_2 in a ratio of about 4:1 is used in the laser tube. The N_2 molecules are excited by collisions with electrons, and then transfer their energy to the upper level of the CO_2 molecules. This gives population inversion in much the same sort of way as for the HeNe mixture.

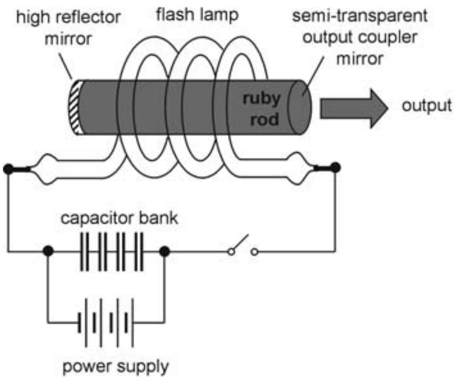
SOLID STATE LASERS

Ruby

Ruby lasers have historical importance because they were the first successful laser to operate. Ruby consists of Cr^{3+} ions doped into crystalline Al_2O_3 (sapphire) at a typical concentration of around 0.05% by weight. The Al_2O_3 host crystal is colourless. The light is emitted by transitions of the Cr^{3+} impurities. Ruby is a *three- level laser*. (See Laser Physics 10). The level scheme is shown below.



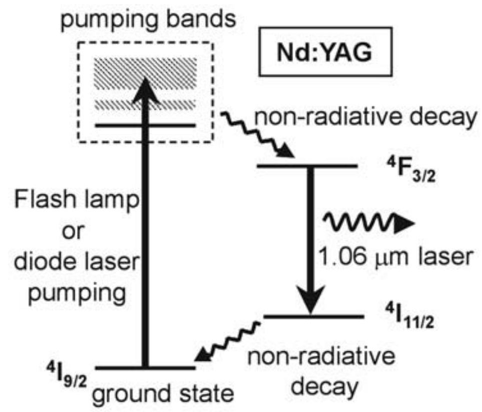
Ruby has strong absorption bands in the blue and green spectral regions (hence the red colour: *ruber* means “red” in Latin.) Electrons are excited to these bands by a powerful flashlamp. (See diagram to the left.) These electrons relax rapidly to the upper laser level by non-radiative transitions in which phonons are emitted. This leads to a large population in the upper laser level. If the flashlamp is powerful enough, it will be possible to pump more than half of the atoms from the ground state (level 0) to the upper laser level (level 2). In this case, there will then be population inversion between level 2 and level 0, and lasing can occur if a suitable cavity is provided. The laser emission is in the red at 694.3nm.



The diagram to the left shows a typical arrangement for a ruby laser. The crystal is inserted inside a powerful flashlamp. Water-cooling prevents damage to the crystal by the intense heat generated by the lamp. Mirrors at either end of the crystal define the cavity. Reflective coatings can be applied directly to the end of the rod as shown, or external mirrors can be used (not shown). The lamps are usually driven in pulsed mode by discharge from a capacitor bank. The pulse energy can be as high as 100 J per pulse. This is because the upper laser level has a very long lifetime (3 ms) and can store a lot of energy.

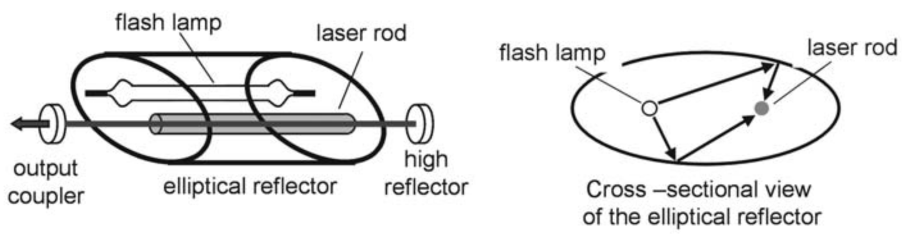
Nd:YAG & Nd:glass

Neodymium ions form the basis for a series of high power solid state lasers. In the two most common variants, the Nd^{3+} ions are doped into either Yttrium Aluminium Garnet (YAG) crystals or into a phosphate glass host. These two lasers are known as either Nd:YAG or Nd:glass. The main laser transition is in the near infrared at 1.06 μm. The wavelength does not change much on varying the host.



The diagram to the left shows the level scheme for the Nd^{3+} lasers, which are *four-level lasers*. Electrons are excited to the pump bands by absorption of photons from a powerful flashlamp or from a diode laser operating around 800 nm. The electrons rapidly relax to the upper laser level by phonon emission. Lasing then occurs on the $^4F_{3/2} \rightarrow {}^4I_{11/2}$ transition. The electrons return to the ground state by rapid non-radiative decay by phonon emission.

The diagrams below show the cavity arrangement in a flashlamp-pumped system. The rod and lamp are positioned at the foci of an elliptical reflector. This ensures that most of the photons emitted by the lamp are incident on the rod to maximize the pumping efficiency. Mirrors at either end of the rod provide the optical cavity. The laser can either be operated in pulsed or continuous mode.

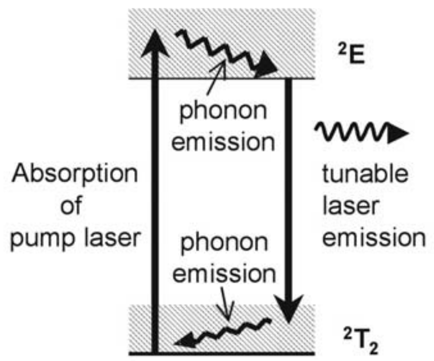


As with the ruby laser, the lifetime of the upper laser level is long: 0.2 – 0.3 ms, depending on the host. This long lifetime, which is a consequence of the fact that the laser transition is E1-forbidden, allows the storage of large amounts of energy. Continuous Nd:YAG lasers can easily give 20–30 W, while pulsed versions can give energies up to 1 J in 10 ns. The pulse energies possible from Nd:glass lasers are even higher, although they can only operate at lower repetition rates. The Lawrence Livermore Lab in California uses Nd:glass lasers for fusion research. The pulse energy in these systems is ~10 kJ. With pulse durations in the 10 ns range, this gives peak powers of 10^{12} W.

Nd lasers are extensively used in industry for cutting applications, and in medicine for laser surgery. They are very rugged and can be used in extreme conditions (eg onboard military aircraft). Frequency-doubled Nd:YAG lasers are now commonly used for pumping tunable lasers. (See below)

Ti:sapphire

Titanium-doped sapphire lasers represent the current state-of-the-art in tunable lasers. The level scheme is shown below. The active transitions occur in the Ti^{3+} ion. This has one electron in the 3d shell. In the octahedral environment of the sapphire (Al_2O_3) host, the crystal field splits the five *m* levels of the 3d shell into a doublet and a triplet. These are labelled as the ^2E and $^2T_2$ states in the diagram to the left[4]. The electron-phonon coupling in Ti:sapphire is very strong, and the ^2E and $^2T_2$ states are broadened into bands. The absorption of the ^2E band peaks in the green-blue spectral region, and thus can be pumped by the 488 nm and 514 nm lines of an argon ion laser. Alternatively, a frequency-doubled Nd:YAG laser operating at (1064/2 = 532 nm) can be used.

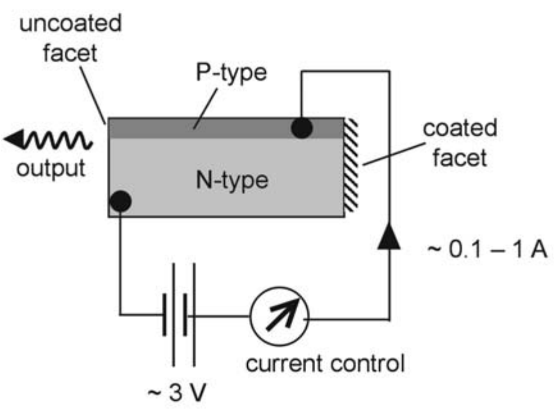


Electrons excited into the middle of the ^2E band rapidly relax by phonon emission to the bottom of the band. Laser emission can then take place to anywhere in the $^2T_2$ band. The electrons finally relax to the bottom of the $^2T_2$ band by rapid phonon emission.

The fact that tuning can be obtained over the entire $^2T_2$ band is a very useful feature because it means that the laser wavelength can be chosen at will. Lasing has in fact been demonstrated all the way from 690 nm to 1080 nm i.e. over nearly 400 nm. This is why it makes sense to use one laser to pump another: we convert a fixed frequency laser such as the argon ion or frequency-doubled Nd:YAG into a tunable source. Energy conversion efficiencies up to 25% are possible.

The broad emission band width is also ideal for making short pulse mode-locked lasers. (See Laser Physics 11). The shortest pulses that can be produced are given by $\Delta\nu\Delta t \gtrsim 1/2\pi$. With such a broad emission band, it has been possible to generate pulses as short as 10 fs.

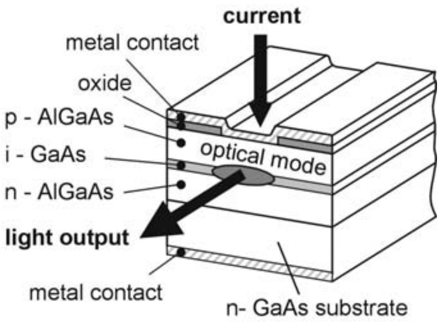
Semiconductor Diode Lasers



Semiconductor diode lasers are by far the most common types of lasers. They are used in laser printers, compact disc players, laser pointers, and optical fibre communication systems. The laser consists of a semiconductor p-n diode cleaved into a small chip, as shown to the left. Electrons are injected into the n-region, and holes into the p-region. The electrons and holes recombine in the active region at the junction and photons are emitted. The laser cavity is formed by using the cleaved facets of the chips. The refractive index of a typical semiconductor is in the range 3–4, which gives about 30%

[4] This notation may be familiar to the Chemical Physicists. The letters are abbreviations for German words. “E” and “T” label doublet and triplet states. The superscript of 2 refers to the spin degeneracy. Thus these two states contain (2×2 + 2×3=10) levels, as we would expect for the 3d states. The subscript of 2 on the triplet state indicates that it has a particular symmetry.

reflectivity at each facet. This is enough to support lasing, even in crystals as short as ~1 mm, because the gain in the semiconductor crystal is very high. A highly reflective coating is often placed on the rear facet to prevent unwanted losses through this facet and hence reduce the threshold. The energy of the photons emitted is equal to the bandgap of the semiconductor. The drive voltage must be at least equal to E_g/e, where E_g is the band gap, and e is the electron charge.



The semiconductor must have a *direct band gap* to be an efficient light emitter. Silicon has an indirect band gap, and is therefore no good for laser diode applications. The laser diode industry is based mainly on the compound semiconductor GaAs, which has a direct band gap at 1.4 eV (890 nm). A typical design of a GaAs diode laser is shown to the left. By using alloys of GaAs, the band gap can be shifted into the red spectral region for making laser pointers, or further into the infrared to match the wavelength for lowest losses in optical fibres (1500 nm). Very recently the short wavelength emission limit has been extended into the blue by using the wide band gap III-V semiconductor GaN in the active region.

The power conversion efficiency of electricity into light in a diode laser is very high, with figures of 25% typically achieved. This compares with typical efficiencies < 0.1% in gas lasers. Since the laser chips are so small, it is possible to make high power diode lasers by running many GaAs chips in parallel. Laser power outputs over 20W can easily be achieved in this way. These high power laser diodes are very useful for pumping Nd:YAG lasers.

<u>Reading</u>

Demtröder, *Atoms, Molecules and Photons* (2006): § 8.4
Wilson and Hawkes, *Optoelectronics* 3rd edition (1998): § 5.10.1 – 5.10.3
Smith and King, Optics and Photonics, §15.2, 15.9, and chapter 16.
Hecht, *Optics* 4th edition (2002): §13.1
Yariv, *Optical Electronics in Modern Communications* 5th edition (1997): chapter 7 *
Silfvast, *Laser Fundamentals* (1996): chapters 13 – 14 *
Svelto, *Principles of Lasers*, 4th edition (1998): chapters 9 – 10 *

* more advanced treatments, some of which is beyond the level of this course.