# QM4226
# QUANTUM THEORY &
# HOMEWORK PROBLEMS 4

These questions form part of the continuous assessment of the course. To be handed in by ...

1. Construct the state:
   $|j_1 j_2 JM >= |3/2\ 1\ 5/2\ 1/2 >$ using the eigenstates $|j_1 m_1 > |j_2 m_2 >$, of the one-particle operators, using the tables of Clebsch-Gordan coefficients.

2. Repeat for $|1/2\ 1\ 3/2\ 1/2 >$.

3. Work out $< J_{1Z} >$ for the states in (1) and (2).

4. A quantum state is in a superposition of the orthonormal eigenstates:
   $\chi = \sqrt{\frac{2}{3}}|3/2\ 1\ 5/2\ 1/2 > +\sqrt{\frac{1}{3}}|1/2\ 1\ 3/2\ 1/2 >$.
   Work out $< \mathbf{J_1}.\mathbf{J_2} >$.

5. For the same state, work out $< J_{1+}J_{2-} + J_{2+}J_{1-} >$.

6. Two electrons interact via a potential: $V(\mathbf{r}) = A\sigma_{\mathbf{1}}.\sigma_{\mathbf{2}}$
   The electrons are in a orthonormal spatial eigenstate, and a spin singlet state, ie $\Psi(\mathbf{r})\chi = R_{ns}(r)Y_{00}\sqrt{\frac{1}{2}}(\alpha_1\beta_2 - \beta_1\alpha_2)$ Work out the interaction energy $< V(\mathbf{r}) >$.

7. $\mathbf{n} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$ is a vector of unit length, representing the orientation of the proton-proton axis. Show that:

   $$(\mathbf{S}.\mathbf{n}) = \frac{1}{2}[S_+ e^{-i\phi}\sin\theta + S_- e^{i\phi}\sin\theta + 2S_z\cos\theta]$$

   Represent ($\mathbf{S}.\mathbf{n}$ as a matrix, using the eigenstates of $\mathbf{S_z}$.

   Work out the expectation value of $(\mathbf{S}.\mathbf{n})^2$ if a quantum system is in the spin-state $\chi = \sqrt{\frac{2}{3}}|\alpha > +\sqrt{\frac{1}{3}}|\beta >$.

8. Show that $R_\pi e^{-i\beta_n\sigma_z t}R_\pi = e^{+i\beta_n\sigma_z t}$ Eq.(4.92) in the lecture notes.

9. Show that $e^{+ia\sigma_z}(\sigma_x\cos\omega t + \sigma_y\sin\omega t)e^{-ia\sigma_z} = \sigma_x$ if $a = \frac{\omega t}{2}$ ie Eq.(4.84) in the lecture notes.