# QM4226
# QUANTUM THEORY &
# HOMEWORK PROBLEMS 3

These questions form part of the continuous assessment of the course. To be handed in by ...

1. For a particle in a potential with $V(x) = -V_0(1 - \frac{|x|}{a})$ for $|x| \leq a$ and $V(x) = 0$ elsewhere and if $\frac{ma^2V_0}{\hbar^2} = 50$, use the WKB approximation to show

   (a) there are 4 bound states

   (b) their energies are $E/V_0 = -.76, -.5, -.3, -.12$.

   Hint: For most WKB questions 1) Sketch the potential well or barrier; are energies likely to be negative or positive? Work out classical turning points 2) work out phase integral between turning points. 3)Apply any quantisation conditions (eg One hard wall? Two? None?). Here you will use the integral $\int(1-y)^{1/2}dy = -2/3(1-y)^{3/2}$.

2. A particle moving in one dimension with energy $E$ is incident on a potential

$$V(x) = \frac{V_0}{|x|}$$

   with $V_0 > 0$. Show that the transmission coefficient, $T_{WKB}$, in the $WKB$ approximation satisfies

$$\ln T_{WKB} \propto \frac{-V_0}{E^{1/2}}$$

   You can solve the integral $\int(c/x - 1)^{1/2}dx$ with the substitution $x = c\cos^2\theta$.

3. Consider a very simplified model for the fusion of two protons, whereby their sizes are ignored and they interact via the Coulomb potential

$$V(r) = \frac{e^2}{4\pi\epsilon_0 r}$$

   where $r$ is the separation between protons. Obtain an estimate for the probability $T = \exp -2\lambda$ that one proton, of energy $E$ incident on the other, will penetrate the Coulomb barrier to the centre $r = 0$.