# QM4226
# QUANTUM THEORY HOMEWORK 1

To be handed in by ...

1. (a) A bound quantum system has a complete set of orthonormal energy eigenfunctions $u_n$ with non-degenerate eigenvalues $E_n$. The operator $Q$ corresponds to some other observable and is such that:

$$Qu_1 = u_2; \quad Qu_2 = u_1; \quad Qu_n = 0 \ ( \ if \ n \geq 3)$$

   (b) Show that $\phi = u_1 \pm u_2$ and $\phi = u_i \ i > 3$ form a set of eigenfunctions of $Q$ and give them in normalised form. (In fact they can be shown to form a *complete* set, but you are not asked to show that).

   (c) If the observable corresponding to $Q$ is measured and is found to have eigenvalue 1, what is the expectation value of the energy in the resulting state?

2. The operators $x$ and $p_x$ corresponding to position and momentum, respectively, satisfy the fundamental commutator:

$$[x, p_x] = i\hbar$$

if $C_n = [x^n, p_x]$, show that $C_n$ obeys the recurrence relation,

$$C_n = xC_{n-1} + i\hbar x^{n-1}$$

Hence, by repeated application show that:

$$[x^n, p_x] = i\hbar n x^{n-1}$$

You can use $[AB, C] = A[B, C] + [A, C]B$.

3. Verify whether

$$\mathrm{e}^{A+B} \simeq \mathrm{e}^{A} \ \mathrm{e}^{B} \ \mathrm{e}^{-[A,B]/2}$$

and

$$\mathrm{e}^{A+B} \simeq \mathrm{e}^{A/2} \ \mathrm{e}^{B} \ \mathrm{e}^{A/2}$$

where $A$ and $B$ are quantum operators multiplied by an arbitrarily small time increment.

4. The orthonormal eigenstates $|n>$ of a one-dimensional harmonic oscillator of angular frequency $\omega$ and energy $E_n = (n + 1/2)\hbar\omega$, n=0,1,2...may be considered as states containing $n$ quanta each of energy $\hbar\omega$. Consider the normalised state $|\alpha>$ given by:

$$|\alpha >= e^{-|\alpha^2|/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n>$$

Where $\alpha$ is some parameter (generally complex) defining the state. Find:
(a) The probability that the state $|\alpha>$ contains $k$ quanta each of energy $\hbar\omega$.
(b) The average number of quanta in state $|\alpha>$.
*NB* note that $|\alpha>$ is sometimes termed a *coherent state*. In a position representation it would correspond to the gaussian wavepackets studied in Lec. 2.